LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **APRIL 2023**

PMT1MC06 – PROBABILITY THEORY AND RANDOM PROCESSES

Date: 05-05-2023 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

SECTION A

Answer ALL the Questions

1.			**(5 x 1 = 5)**
a)	State Markov's inequality.	K1	CO1
b)	With usual notations, recall the partial correlation coefficient between x_1 and x_2 with respect to x_3.	K1	CO1
c)	List down the four properties of a good estimator.	K1	CO1
d)	Describe the critical region in testing the statistical hypothesis.	K1	CO1
e)	Define co-variance stationary in random process.	K1	CO1
2.			**(5 x 1 = 5)**
a)	Identify which of the following is true? (i) $E(XY)^2 = E(X^2)E(Y^2)$ (ii) $E(XY)^2 \geq E(X^2)E(Y^2)$ (iii) $E(XY)^2 \leq E(X^2)E(Y^2)$ (iv) None	K2	CO1
b)	Tell the person who introduced the concept of rank regression? (i) Fisher (ii) Pearson (iii) Spearman (iv) Galton	K2	CO1
c)	If T is consistent estimator of θ, then T^2 is identified as (i) Consistent estimator (ii) Inconsistent estimator (iii) Normal estimator (iv) Poisson estimator	K2	CO1
d)	The power of test is estimated using the probability of (i) Type I error (ii) Type II error (iii) Level of significance (iv) None	K2	CO1
e)	If the future values of any sample function can be predicated from the past values, then the random process is identified as (i) Stochastic (ii) Deterministic (iii) Non-deterministic (iv) None	K2	CO1

SECTION B

Answer any THREE of the following **(3 x 10 = 30)**

3.	State and prove Chebyshev's inequality.	K3	CO2
4.	Ten teams participated in a variety event conducted by Loyola College were ranked by the three judges Vijay, Ajith and Surya in the following order:	K3	CO2

Ranks by Vijay	1	6	5	10	3	2	4	9	7	8
Ranks by Ajith	3	5	8	4	7	10	2	1	6	9
Ranks by Surya	6	4	9	8	1	2	3	10	5	7

Using rank correlation method, analyse which pair of judges has the nearest approach to common likings in variety event.?

5.	State and prove the sufficient conditions for consistency.	K3	CO2
6.	State and prove Neyman-Pearson Lemma.	K3	CO2
7.	Show that the random process $X(t) = A\cos(\omega t + \theta)$ is wide-sense stationary, where A and ω are constants and θ is uniformly distributed on the interval $(0,2\pi)$.	K3	CO2

SECTION C

Answer any TWO of the following **(2 x 12.5 = 25)**

8.	State and prove weak law of large numbers. Also, examine whether the law of large numbers holds for the sequence $\{X_k\}$ of independent random variables defined as follows: $P[X_k = \pm 2^k] = 2^{-(2k+1)}$; $P[X_k = 0] = 1 - 2^{-2k}$.	K4	CO3
9.	Explain minimum variance unbiased estimator for $\gamma(\theta)$, and hence show that it is always unique.	K4	CO3
10.	Let p be the probability that a coin will fall head in a single toss in order to test $H_0: p = \frac{1}{2}$ against $H_1: p = \frac{3}{4}$. The coin is tossed 5 times and H_0 is rejected, if more than 3 heads are obtained. Calculate the values of α, β, level of significance and power of the test.	K4	CO3
11.	Explain random telegraph signal process and derive any two of its properties.	K4	CO3

SECTION D

Answer any ONE of the following **(1 x 15 = 15)**

12.	Estimate the Pearson's coefficient of correlation between advertisement cost and sales as per the data given below.	K5	CO4

Cost in Thousands	39	65	62	90	82	75	25	98	36	78
Sales in Lakhs	47	53	58	86	62	68	60	91	51	84

Interpret the result with business industry.

13.	Summarize the four classes of random processes with suitable real-life examples.	K5	CO4

SECTION E

Answer any ONE of the following **(1 x 20 = 20)**

14.	(a)	Prepare an estimation for the cost of a project with minimum five parameters using the concept of estimation theory. In addition, estimate the maximum likelihood estimators from the random sampling of normal population $N(\mu, \sigma^2)$, for (i) μ when σ^2 is known (ii) σ^2 when μ is known (iii) the simultaneous estimation of μ and σ^2. **(12 marks)**	K6	CO5
	(b)	Find the nature of the states of the Markov chain with the transition probability matrix $P = \begin{matrix} \\ 0 \\ 1 \\ 2 \end{matrix} \begin{matrix} \begin{matrix} 0 & 1 & 2 \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix} \end{matrix}$ **(8 marks)**		
15.		Create a social science-related dataset and explain how regression lines can be used to predict an unknown data.	K6	CO5

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