LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **APRIL 2023**

PMT2MC04 – COMPLEX ANALYSIS

Date: 06-05-2023 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

SECTION A – K1 (CO1)

	Answer ALL the questions	**(5 x 1 = 5)**
1.	**Answer the following**	
a)	What is the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{z^n}{n!}$.	
b)	Define zeros of an analytic function $f(z)$ defined on an open set G of multiplicity $m \geq 1$.	
c)	What do you mean by a closed rectifiable curve γ_0 in G homotopic to zero?	
d)	Define a convex set with an example.	
e)	State Functional equation.	

SECTION A – K2 (CO1)

	Answer ALL the questions	**(5 x 1 = 5)**
2.	**Choose the correct answer for the following**	
a)	The radius of convergence of the power series $\sum_{n=0}^{\infty} a_n(z-a)^n$is (i)) $\frac{1}{R} = \lim sup\|a_n\|^{\frac{1}{n}}$ (ii) $R = \lim sup\|a_n\|^{\frac{1}{n}}$ (iii) $R = \lim sup\|a_n\|^n$ (iv) $\frac{1}{R} = \lim sup\|a_n\|^n$	
b)	If $p(z) = z^n + a_{n-1}z^{n-1} + \cdots a_0$ then $\lim_{n\to\infty} p(z) =$ (i) 0 (ii) 1 (iii) ∞ (iv) n	
c)	If $f: G \to C$ is an analytic function and γ is a closed rectifiable curve such that $\gamma \sim 0$ then$\int_\gamma f =$ (i) nonzero (ii) 2 (iii) 1 (iv) 0	
d)	$E_0(z) =$ (i)1 (ii) $1 - z$ (iii) $1 + z$ (iv) $1 + 2z$	
e)	For $z \neq 0, -1, \dots \infty$ (i) $\gamma(z) = \lim_{n\to\infty} \frac{(n-1)!n^z}{z(z+1)(z+2)\dots(z+n)}$ (ii) $\gamma(z) = \lim_{n\to\infty} \frac{(n)!n^z}{z(z+1)(z+2)\dots(z+n)}$ (iii) $\gamma(z) = \lim_{n\to\infty} \frac{(n-1)!n^z}{z(z+1)(z+2)\dots(z+n+1)}$ (iv) $\gamma(z) = \lim_{n\to\infty} \frac{(n)!n^z}{(z+1)(z+2)\dots(z+n)}$	

SECTION B – K3 (CO2)

	Answer any THREE of the following	**(3 x 10 = 30)**
3.	Prove $\int_0^{2\pi} \frac{e^{is}}{e^{is}-z} ds = 2\pi \ if \ \|z\| < 1$.	
4.	State and prove the fundamental theorem of algebra.	
5.	Let $Rez_n > -1$. Prove that the series $\sum log(1+z_n)$ converges absolutely if and only if the series $\sum z_n$ converges absolutely.	
6.	Prove that a differentiable function on $[a, b]$ is convex if and only if f' is increasing.	
7.	State and prove the Gauss's formula.	

SECTION C – K4 (CO3)	
	Answer any TWO of the following **(2 x 12.5 = 25)**
8.	State and prove Cauchy's integral formula and apply to evaluate $\int_{\gamma} \frac{1}{z-a} dz$ where $\gamma = a + re^{it}, 0 \leq t \leq 2\pi$.
9.	State and prove Morera's theorem.
10.	Prove Schwarz's lemma and apply it to prove the following: Let $g: D \to D$ be analytic and $g(0) = 0$. Let $h(z) = \frac{g(z)}{z}, z \neq 0$ $= g'(0), z = 0$ (i) $h(z)$ is analytic in D. (ii) $h(D) \subseteq \overline{D}$ (iii)$\lvert g(1/2) \rvert \leq 1/2$
11.	If $\gamma_0$and $\gamma_1$are two closed rectifiable curves in G such that $\gamma_0 \sim \gamma_1$explain how$\int_{\gamma_0} f = \int_{\gamma_1} f$.
SECTION D – K5 (CO4)	
	Answer any ONE of the following **(1 x 15 = 15)**
12.	Let *f* and *g* be analytic on a region *G*. Prove that $f \equiv g$ if and only if $\{z \in C: f(z) = g(z)\}$ has a limit point in G. If $f: C \to C$ is an entire function and g(z) is defined by $g(z) = f(z) - f(z+1)$, $f\left(\frac{1}{n}\right) = 0, and\ f\left(\frac{1}{n}\right) = f\left(\frac{1}{n} + 1\right)$, for all positive integer value of n, Can we say that *f* and *g* are constant? Justify.
13.	Prove the weierstrass factorization theorem and evaluate the factorization of sine function.
SECTION E – K6 (CO5)	
	Answer any ONE of the following **(1 x 20 = 20)**
14.	Is there an analytic function f on $B(0;1)$ such that $\lvert f(z) \rvert < 1$for $\lvert z \rvert < 1$, $f(0) = 1/2$ and$f'(0) = 3/4$? If so find such an *f*. Is it unique? Prove the supporting result.
15.	Let *G* be a simply connected region which is not the plane and let $a \in G$. Construct a unique analytic function $f: G \to C$ having the properties: (a)$f(a) = 0$ and $f'(a) > 0$ (b) f is one-one (c) $f(G) = \{z: \lvert z \rvert < 1\}$

$$$$$$$