



Date: 09-11-2016

Dept. No.

Max. : 100 Marks

Time: 01:00-04:00

**Part A**

**Answer all Questions:**

**( 10 x 2 = 20 )**

1. Write the  $n^{\text{th}}$  derivative of  $y = \sin(ax + b)$ .
2. Show that the parabola  $y^2 = 4ax$ , the sub tangent at any point is double the abscissa and the subnormal is constant.
3. Define D' Alembert's ratio test.
4. Write the expansion of  $\log(1-x)$ .
5. Find the value of  $L^{-1} \left[ \frac{1}{s(s+a)} \right]$ .
6. Find  $L [t^2 + 2t + 3]$ .
7. Define Skew Hermitan Matrix with example.
8. Find characteristic equation of  $A = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$
9. What is the chance that the leap year selected at random will contain 53 Sundays?
10. Define Rank correlation.

**Part B**

**Answer any FIVE questions:**

**( 5 x 8 = 40 )**

11. Find the angle of intersection of the coordinates  $r = a(1 + \cos \theta)$  and  $r = b(1 - \cos \theta)$ .
12. Find Maxima minima of the function  $2x^3 - 3x^2 - 36x + 10$ .
13. Test the convergence of the series  $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \dots + \infty$ .
14. Find sum to infinity of the series  $1 + \frac{3}{4} + \frac{3}{4} \frac{5}{8} + \frac{3}{4} \frac{5}{8} \frac{7}{12} + \dots$
15. Find the transform of the rectangular wave whose function is given as:

$$f(x) = \begin{cases} 1, & 0 \leq t \leq b \\ -1, & b \leq t \leq 2b. \end{cases}$$

16. Find the value of  $L^{-1} \left[ \frac{s}{s^2 + 2s + 5} \right]$ .

17. Verify Cayley – Hamilton theorem for the matrix  $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$  and find  $A^{-1}$ .

18. The rank of the same 16 students in mathematics and physics are as follows. Two numbers with in the bracket denote the ranks of the student in mathematics and physics:  
 (1,1), (2,10), (3,3), (4,4), (5,5), (6,7), (7,2), (8,6), (9,8), (10,11), (11,15), (12,9), (13,14), (14,12), (15,16), (16,13). Calculate the rank of the correlation co-efficient for the proficiencies of this group in mathematics and physics.

**Part C**

**Answer any TWO questions:**

**( 2 × 20 = 40 )**

19. (a) Find the nth differential coefficient of  $x^2 \log x$ .

(b) If  $y = (x + \sqrt{1+x^2})^m$  then prove that  $(1+x^2)y'' + xy' - m^2y = 0$  and

$$(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0.$$

(6+14)

20. (a) Solve the equation  $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 4e^{-t}$  given  $y(0) = 0$  and  $y'(0) = 0$  using Laplace transforms.

(b) Evaluate  $\int_0^{\infty} \frac{e^{-t} - e^{-2t}}{t} dt$ .

(12+8)

21. (a) Find the Eigen values Eigen vectors of the matrix  $\begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$ .

(b) Find the inverse of  $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$ .

(10+10)

22. (a) Calculate the mean and standard deviation for the following table giving the age distribution of 542 members:

Age in years	20-30	30-40	40-50	50-60	60-70	70-80	80-90
Number of members	3	61	132	153	140	51	2

(b) Sum the series  $1 + \frac{1+3}{2!} + \frac{1+3+3^2}{3!} + \dots + \infty$ .

(10+10)

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