



Date: 09-11-2016
Time: 01:00-04:00

Dept. No.

Max. : 100 Marks

PART A

Answer all the questions:

(10 X 2 = 20)

1. If $f(x) = (4x - 1)(x - 5)$, find the values of $f(3)$ and $f\left(\frac{1}{2}\right)$.
2. Differentiate $\frac{x^3}{3x-2}$ with respect to x .
3. For what value of x is $6x^3 - 2x^2 + 7x - 4$ a decreasing function?
4. Find the point of inflexion on $y = x^3 - 9x^2 + 7x - 6$.
5. Using Maclaurin's series, expand $\tan x$ as an infinite series.
6. Find the first order partial differential coefficients of $u = \cos(7x + 4y)$.
7. Integrate $x^2 e^x$ with respect to x .
8. Evaluate $\int \frac{dx}{4+9x^2}$.
9. Write any two properties of definite integrals.
10. Find $\int_1^2 (2x^3 + x - 4) dx$.

PART B

Answer any FIVE questions:

(5 X 8 = 40)

11. (a) If $y = \frac{(x+3)}{(x+2)}$, find $\frac{dy}{dx}$.
(b) Prove that the tangents to the curve $y = x^2 - 5x + 6$ at the points (2,0) and (3,0) cut at right angles. (3+5)
12. Show that the curve $y = \frac{6x}{x^2+3}$ has three points of inflexion.
13. Using mean value theorem, determine c , lying between a and b , when
(i) $f(x) = x^3 - 2x^2, a = 2, b = 5$
(ii) $f(x) = x^3 + x, a = 1, b = 2$.
14. If $u = \log(x^2 + y^2 + z^2)$, prove that $x \frac{\partial^2 u}{\partial y \partial z} = y \frac{\partial^2 u}{\partial z \partial x} = z \frac{\partial^2 u}{\partial x \partial y}$.
15. Integrate $x^2 \cos 3x$ with respect to x .
16. Evaluate $\int \frac{x}{x^2+x+1} dx$.
17. Prove that $\int_0^{\frac{\pi}{2}} \log \sin x dx = \frac{\pi}{2} \log \left(\frac{1}{2}\right)$.
18. Evaluate $\int (x^2 + y^2) dx dy$ over the region for which $x, y \geq 0$ and $x + y \leq 1$.

PART C

Answer any TWO questions:

(2 X 20 = 40)

19. (a) If $f(x) = x^3 + x^2 + x - 1$, simplify $f(x + 1) - 3f(x) + 2f(x - 1)$
(b) If $y = \sin x \sin 2x \sin 3x$, find $\frac{dy}{dx}$.
(c) Differentiate $x^{(\log x)^2}$ with respect to $(x \log x)(\log \log x)$

(7+6+7)

20. (a) Find the maximum and minimum values of the function $y = x^3 - 18x^2 + 96x + 4$.
 (b) Prove that $\log(1 + x + x^2) = x + \frac{1}{2}x^2 - \frac{2}{3}x^3 + \frac{1}{4}x^4 + \dots$. (10+10)
21. (a) Verify Euler's theorem when $u = x^3 - 3x^2y + 3xy^2 + y^3$.
 (b) Prove that $\int_0^{\frac{\pi}{4}} \log(1 + \tan\theta) d\theta = \frac{\pi}{8} \log 2$. (10+10)
22. (a) Evaluate $\int \frac{dx}{(x+1)\sqrt{x^2+x+1}}$.
 (b) By transforming into polar coordinates, evaluate $\iint \frac{x^2y^2}{x^2+y^2} dx dy$ over the annular region between the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$ ($b > a$). (10+10)
