LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2016**

MT 1502 – ALGEBRA AND CALCULUS - I

Date: 05-11-2016 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART-A

Answer ALL questions: **(10× 2 = 20)**

[1] Find the nth differential coefficient of $(2x+3)^m$.

[2] Show that the polar sub tangent of the curve $r = e^{\theta \cot \alpha}$ is $r \tan \alpha$.

[3] Write down the condition for maxima or minima of two variables.

[4] Write the steps used in Lagrange's method of undetermined multiplier to find the minimum or maximum value of $f(x,y,z)$ subject to the condition $\varphi(x,y,z)=0$.

[5] Define curvature.

[6]Write down the pedal equation of a curve.

[7]Form a rational cubic equation which have the roots $1, 3-\sqrt{-2}$.

[8]Find the sum and the product of the four roots of the equation $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$.

[9] Find the imaginary roots of the equation $x^7 + 8x^5 - x + 9 = 0$.

[10] If α, β, γ are the roots of the equation $x^3 + qx + r = 0$, find the equation whose roots are $5\alpha, 5\beta, 5\gamma$.

PART-B

Answer any FIVE **(5× 8 = 40)**

[11] Find (i) $D^n(\cos^3 x)$ (ii) $D^n(e^x \sin x)$ (4+4)

[12]Find the angle of intersection of the cardioids $r = a\,(1+\cos\theta)$ and $r = b\,(1-\cos\theta)$.

[13] Discuss the maxima or minima of the function $f = y^2 + 2yx^2 + 4x - 3$.

[14]Find the minimum value of $f = x^2 + y^2 + z^2$ when $x + y + z = 3a$.

[15] Prove that the radius of curvature at any point of the cycloid $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$ is $4a\cos\frac{\theta}{2}$.

[16]Solve the equation $x^4 + 4x^3 + 5x^2 + 2x - 2 = 0$ which one root is $-1+\sqrt{-1}$.

[17]Find $\frac{1}{\alpha^5} + \frac{1}{\beta^5} + \frac{1}{\gamma^5}$,when α, β, γ are the roots of the equation $x^3 + 2x^2 - 3x - 1 = 0$.

[18] Show that the roots of the equation $x^3 + px^2 + qx + r = 0$, are in A.P if $2p^3 - 9pq + 27r = 0$.

PART-C

Answer any TWO: **(2× 20 = 40)**

[19] (i) If $y = e^{a\sin^{-1}x}$, prove that $(1-x^2)y_2 - xy_1 - a^2 y = 0$ .Hence show that

$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 + a^2)y_n = 0$. (10+ 10)

(ii) Find the maximum or minimum values of $2(x^2 - y^2) - x^4 + y^4$.

[20] (i) Find the evolute of the parabola $y^2 = 4ax$ at the time 't'. (12+ 8)

(ii) Find the asymptotes of $x^3 + 2x^2 y - x \ y^2 - 2y^3 + 4y^2 \ + 2xy + y - 1 = 0$.

[21] (i) If the sum of the two roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$ equals the sum the other two, prove that $p^3 + 8r + = 4pq$. (10+ 10)

(ii) Solve the equation $6x^5 - x^4 + 43x^3 + 43x^2 + x - 6 = 0$.

[22] (i) Decrease the roots of the equation $x^5 - 6x^2 - 4x + 5 = 0$ by 3. (6+14)

(ii)Find the real root of $x^3 - 7x + 7 = 0$ correct to two place of decimals using Horner's method.

################