# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**M.Sc.** DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – NOVEMBER 2016

**MT 1819 - PROBABILITY THEORY & STOCHASTIC PROCESSES**

Date: 12-11-2016 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

**PART –A**

**Answer ALL questions** **10 * 2 = 20**

1. Define distribution function of a random variable.
2. What is meant by equally likely events?.
3. Write the sample space for tossing two fair coins simultaneously.
4. Find $E(X)$, if X is an exponential random variable.
5. Write the probability distribution function of exponential distribution.
6. Define convergence in distribution.
7. Write any two properties of normal distribution.
8. Define a consistent estimators.
9. How do you understand Markov chain.
10. Define Chebyshev's inequality.

**PART-B**

**Answer any FIVE questions** **5 * 8 = 40**

11. Two cards are drawn from a pack of 52 cards with replacement. Find the probability that (i) both are King, (ii) first is King and the second is Queen, (iii) one is King and other is Queen.
12. State and prove Baye's theorem.
13. Find the moment generating function of exponential distribution and hence find its mean and variance.
14. A random variable X has the following probability function:

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| P(x) | 0 | k | 2k | 2k | 3k | [illegible] | $2k^2$ | $7k$ [illegible] |

(i) Find k. (ii) Evaluate $P(X < 6)$, $P(X \geq 6)$, and $P(0 < X < 5)$ (iii) If $P(X \leq a) > \frac{1}{2}$, find the minimum value of a.

15. Let $x_1, x_2, .. , x_n$ be a random sample from a uniform population on $[0, \theta]$. Find a sufficient estimator for $\theta$.
16. Find the maximum likelihood estimator for the parameter $\lambda$ of a poisson distribution on the basis of a sample size *n*.
17. State and prove Rao-Blackwell theorem.
18. If $x \geq 1$, is the critical region for testing $H_0: \theta = 2$ against the alternative $\theta = 1$, on the basis of the single observation from the population $f(x, \theta) = \theta e^{-\theta x}, 0 \leq x < \infty$, obtain the values of type I and type II errors.

**PART –C**

**Answer any TWO questions** **2 * 20 = 40**

19.a) Let $X$ be a continuous random variable with p.d.f given by

$$F(x) = \begin{cases} kx, & 0 \le x < 1 \\ k, & 1 \le x < 2 \\ -kx + 3x, & 2 \le x < 3 \\ 0, & elsewhere \end{cases}$$

(i)Determine the constant $k$, (ii) Determine $F(x)$, the c.d.f and (iii) If $x_1, x_2, x_3$ are three independent observation from $X$, what is the probability that exactly one of these three numbers is larger than 1.5?

b) Let A and B be two events in a sample space S such that $P(A) = \frac{1}{2}$,

$P(\bar{B}) = \frac{5}{8}, P(A \cup B) = \frac{3}{4}$. Find $P(A \cap B)$ and $P(\bar{A} \cup \bar{B})$.

20.a) Find the marginal distribution of X and Y, and the conditional distribution of Y for X=x.

b) Obtain the first and second central moments of Beta distribution of second kind.

21.a) State and prove Cramer-Rao inequality.

b) State and prove Neyman- Pearson lemma.

22. Explain the postulates of a Poisson process, and derive the transition distribution of the process.

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