LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – NOVEMBER 2016

MT 2810 - ALGEBRA

Date: 08-11-2016 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

ANSWER ALL QUESTIONS.

I a) Prove that conjugacy is an equivalence relation.

[OR]

b) If $o(G) = p^2$ where p is a prime number , then prove that G is abelian. **(5)**

c i) If p is a prime number and $p/o(G)$, then prove that G has an element of order p

ii) If p is a prime number, then prove that any group of order $2p$ has a normal subgroup of order p **(10+5)**

[OR]

d i) State and prove Sylow's first theorem.

ii) Let G be a group of order $11^2 13^2$. Discuss about the Sylow subgroups of G. Also prove that G is abelian **.** **(7+8)**

II a) Show that $x^6 + 8x^5 - 16x^4 + 24x^3 - 20x + 10$ is irreducible over rational numbers.

[OR]

b) If $f(x)$ *and* $g(x)$ are primitive polynomials then prove that $f(x)g(x)$ is a primitive polynomial. **(5)**

c i) State and prove the Eisentein criterion.

ii) Prove $x^2 + 1$ is irreducible over the integers mod 7. **(10+5)**

[OR]

d i) State and prove Gauss lemma.

ii) Prove that the polynomial $1 + x + x^2 ... + x^{p-1}$, where p is a prime number is irreducible over the field of rational numbers**.** **(7+8)**

III a) Find the degree of $\sqrt{2} + \sqrt{3}$ over Q .

[OR]

b) Determine the degree of the splitting field of the polynomial $x^4 + 1$ over the field of rational numbers **(5)**

c) Prove that the element $a \in K$ is algebraic over F if and only if $F(a)$ is a finite extension of F**.** **(15)**

[OR]

d) i) Prove that a polynomial of degree n over a field can have at most n roots in any extension field.

ii) Let F be any field and let $p(x) = x^2 + \alpha x + \beta,\ \alpha, \beta \in F$ be in $F[x]$ .Prove that $p(x)$ can be split by any extension of degree 2 of F. **(7 + 8)**

IV a) Express $x_1^2 + x_2^2 + x_3^2$ in the elementary symmetric functions in x_1^2, x_2^2, x_3^2.

[OR]

b) Prove that the fixed field of G is a subfield of K. **(5)**

c) Let F be a field and let $F(x_{1,} \ldots \ldots . x_n)$ be the field of rational functions in $x_{1,} \ldots \ldots . x_n$ over F. Suppose S is the Field of symmetric rational functions. Then prove that

(i) $[F(x_{1,} \ldots \ldots . x_n) : S] = n!$

(ii) $G([F(x_{1,} \ldots \ldots . x_n) : S]) = S_n$

(iii) If $a_{1,} \ldots \ldots . a_n$ are elementary symmetric functions in $x_{1,} \ldots \ldots . x_n$ then $S = F(a_{1,} \ldots \ldots . a_n)$.

(iv) $F(x_{1,} \ldots \ldots x_n)$ is the splitting field over $F(a_{1,} \ldots \ldots . a_n)$ =S of the polynomial $t^n - a_1 t^{n-1} + a_2 t^{n-2} .. + (-1)^n a_n$

[OR]

d) Prove that K is a normal extension of F if and only if K is the splitting field of some polynomial over F. **(15)**

V a) Prove that any two finite fields having the same number of elements are isomorphic. .

[OR]

b) Derive the cyclotomic polynomials $\Phi_3(x)$ and $\Phi_4(x)$. **(5)**

c) Prove that the multiplicative group of nonzero elements of a finite field is cyclic.

[OR]

d) Show that a finite division ring is necessarily a commutative field **(15)**

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