LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – NOVEMBER 2016

MT 2962 - ACTUARIAL MATHEMATICS

Date: 16-11-2016 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

Answer all Questions. All questions carry equal marks.

1. (a) Define distribution and survival functions of the time-until-death random variable $T(x)$ and obtain its expressions in terms of $S(x)$.

(OR)

(b) Define the time-until-death random variable for a person aged x and deferred probabilities. Prove that ${}_{t/u}q_x = {}_u q_{x+t}\ {}_t p_x = {}_{t+u}q_x - {}_t q_x$. (5)

(c) (i) For the current type of refrigerator, it is given that $S(x) = \begin{Bmatrix} 1 & x \le 0 \\ 1-\dfrac{x}{w} & 0 \le x \le w \\ 0 & x > w \end{Bmatrix}$ and l_0^0=20. For a proposed new type, with the same w, the new survival function is $S^*(x) = \begin{Bmatrix} 1 & 0 \le x \le w \\ \dfrac{w-x}{w-5} & 5 < x \le w \end{Bmatrix}$.

Calculate the increase in life expectancy at time 0. (ii) Find A if $\mu_x = A + e^x$ and ${}_{0.50}p_0 = 0.50$.
(iii) If $\mu(x) = 0.001, 20 \le x \le 25$, evaluate ${}_{2/2}q_{20}$. (6+4+5)

(OR)

(d) (i) If S(x) = $1-\frac{x^2}{100}, 0 \le x \le 10$, then calculate (i) $F_X(x)$, (ii) ${}_t p_4$, (iii) ${}_{2/2}q_4$ and (iv) probability density function of $T(4)$.
(ii) Define curtate future life-time random variable $K(x)$ and obtain its probability mass function. Find the distribution of $K(x)$ when $S(x) = 1 - \dfrac{x^2}{100}, 0 \le x \le 10$. Also obtain its expectation e_4.

(6+9)

2. (a) Suppose a survival model is defined by the value of p_x.

x	0	1	2	3	4
p_x	0.9	0.8	0.6	0.3	0

What are the corresponding values of $S(x)$ for $x = 0, 1, 2, 3, 4$ *and* 5.

(OR)

(b) Explain uniform distribution of deaths and hence prove that $l_{x+t} = l_x - t\, d_x$. (5)

(c) Derive an expression for ${}_nD_x$.
(d) An aviary of birds which has a constant intake of 100 new born birds per year experience the following mortality rates:

[illegible]	0	1	2	3	4	5
[illegible]	0.3	0.1	0.2	0.4	0.7	1

(i) What is the expected total number of birds in the aviary at any time?
(ii) What is the expected number living between ages 1 and 4? (10+5)

(OR)

(e) A mortality table has a select period of three years. Find expressions in term of life table functions $l_{[x]+t}$ and l_y for $q_{[50]}$, $2p_{[50]}$, ${}_{2/}q_{[50]}$ and ${}_{2/3}q_{[50]+1}$.

(f) Derive the expression for l_x, d_x, L_x, T_x, e_x and tabulate the values of l_x, d_x, L_x, T_x, e_x where $q_0 = 0.2$, $q_1 = 0.45$, $q_2 = 0.50$, $q_3 = 0.65$, $q_4 = 1$ and taking $l_0 = 100$. (8+7)

3. (a) Find the amount of Rs10, 000/- after 10 years if the rate of interest is 5% per annum payable quarterly.

(OR)

(b) Find the principle, if the amount with compound interest of 5% per annum is 3969 for the period of 2 years.

(5)

(c) If the probability density function of the future life time T is given by $g(t) = \begin{cases} \frac{1}{80}, & 0<t<80 \\ 0 & elsewhere \end{cases}$, then calculate (i) the net single premium at a force of interest δ. (ii) the variance and (iii) the 90th percentile

(d) Give an account of endowment insurance policy. (10+5)

(OR)

(e) (i) Assume that each of 100 independent lives is of age *x*, is subject to a constant force of mortality μ = 0.04 and is insured for a death benefit amount of 10 units, payable at the moment of death. The benefit payments are to be withdrawn from an investment fund earning interest at a rate δ = 0.06. Calculate the minimum amount to be collected at *t*=0, so that the probability is approximately 0.95 that sufficient funds will be on hand to withdraw the benefit payment at the death of each individual.

(ii) Give an account of whole life insurance policy. (9+6)

4. (a) Explain Term –annuity due?

(OR)

(b) In an annuity certain, derive expression for the present value $a_{\overline{n|}}$ of the *n* level payments and the accumulate value $S_{\overline{n|}}$ of $a_{\overline{n|}}$ invested at the time of issue of annuity contract.

(5)

(c) For a 3-year temporary life annuity-due on (30), given $S(x) = 1 - \frac{x}{80}, 0 \le x < 80$ $i = 0.05$ and $Y = \begin{cases} \ddot{a}_{\overline{k+1|}}, k = 0,1,2 \\ \ddot{a}_{\overline{3|}}, \quad k=3,4,5 \end{cases}$, calculate *Var*(*Y*).

(d) Derive whole life annuity due. (10+5)

(OR)

(e) Prove that $\ddot{a}_x = \frac{1 - A_x}{d}$.

(f) Prove that $Var(\ddot{a}_{\overline{k+1|}}) = \frac{{}^2A_x - (A_x)^2}{d^2}$. (8+7)

5. (a) Calculate $\ddot{a}_x$ where it is given that ${}_{10}E_x = 0.40$, ${}_{\overline{10|}}\ddot{a}_x = 7$ and $\ddot{S}_{x:\overline{10|}} = 15$.

(OR)

(b) Define a loss random variable. A fully continuous 10-year term insurance of face amount Rs. 10,000/- has annual premium rate Rs. 100/- and the force of interest is 0.05. Find the value of the issue-date-loss 1) if the death occurs exactly 5 years after issue and 2) if death occurs exactly 15 years after issue.

(5)

(c) For a fully continuous whole life insurance 1 on (x). Calculate $\overline{P}(\bar{A}_x)$ given the following:

(i) Premiums are determine using the equivalence principle.

(ii) $\frac{var[Z]}{var[L]} = 0.36$ and

(iii) $\bar{a}_x = 10$.

(d) If ${}_{k|}q_x = c(0.96)^{k+1}, k = 0,1,2,\ldots$ where c=0.04/0.96 and i=0.06, calculate P_x and Var(L).

(8+7)

(OR)

(e) (i) For (x) you are given the following information:

1) The premium for a 20 −year endowment insurance of 1 is 0.0349.
2) The premium for a 20 −year pure-endowment of 1 is 0.0230.
3) The premium for a 20 −year deferred whole life annuity-due of 1 per year is 0.2037 and is paid for 20 years.
4) All premiums are fully discrete annual benefit premiums.
5) $i = 0.05$.

Calculate the premium for a 20 −payment whole life insurance of 1.

(ii) If ${}_{k|}q_x = c(0.96)^{k+1}, k = 0,1,2,\ldots$ where c=0.04/0.96 and i=0.06, calculate P_x.

(10+5)

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