LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS & STATISTICS**

THIRD SEMESTER – NOVEMBER 2016

MT 3875 / ST 3876 - MATHEMATICAL FINANCE MODELS

Date: 14-11-2016 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

Answer **ALL** questions:

1. (a) Explain the risk neutral probability. **(5)**

(OR)

(b) State and prove doubling rule. **(5)**

(c) (i) An individual who plans to retire in 20 years has decided to put an amount A in the bank at the beginning of each of the next 240 months, after which she will withdraw \$1,000 at the beginning of each of the following 360 months. Assuming a nominal yearly interest rate of 6% compounded monthly, how large does A need to be?

(ii) Find the rate of return from an investment that for an initial payment of 100 yields returns of 60 at the end of each of the first two periods. **(9+6)**

(OR)

(d) State and Prove the Arbitrage theorem. **(15)**

2. (a) Prove that the No arbitrage option cost C is increasing in the initial price s. **(5)**

(OR)

(b) Define Stocks, Shares, call option and put option. **(5)**

(c) Derive the Black Scholes no arbitrage option cost Formula. **(15)**

(OR)

(d) (i) Explain the Delta Hedging Arbitrage Strategy.

(ii) State the put-call option parity formula. **(12+3)**

3. (a) Suppose that a security is presently selling for a price of 60, the nominal rate is 9% (with the unit of time being one year) and the security's volatility is 0.35. Find the no arbitrage cost of a call option that expires in three months and has a strike price 68.

(OR)

(b) Suppose an investor with capital x can invest any amount between 0 and x ; if y is invested, then y is either won or lost, with respective probabilities p and $1-p$. If $p > 1/2$, how much should be invested by an investor having a log utility function? **(5)**

(c) Prove that in call options on dividend paying securities, for each share owned, a fixed amount D is to be paid at time t_d. **(15)**

(OR)

(d) Assuming a General Distribution for the size of a jump, prove that ,

No – arbitrage cost = $E[C(s_t, J(t), K, \sigma, r)] \geq C(s, t, K, \sigma, r)$ and

No arbitrage option cost= $C(s,t,K,\sigma,r)+s_t^2[e^{-\lambda t(1-E[J^2])}-e^{-2\lambda t(1-E[J])}]\frac{1}{2s\sigma\sqrt{2\prod t}}e^{-w^2/2}$ **(15)**

4. (a) Explain in detail, the Expected Value at Risk. **(5)**

(OR)

(b) Derive the formula for β_i in the Capital Assets pricing model. **(5)**

(c) Suppose that three investment projects with the following return functions are available $(i) f_1(x) = \frac{10x}{1+x}, x = 0,1, \ldots$ $(ii)\ f_2(x) = \sqrt{x}, x = 0,1, \ldots$ $(iii) f_3(x) = 10(1 - e^{-x}), x = 0,1 \ldots$ .When we will yield maximum return for we have 5 to invest.

(15)

(OR)

(d) Estimate the volatility parameter when the collection prices follow Geometric Brownian motion.

(15)

5. (a) Explain Asian and Lookback Options.

(OR)

(b) Explain the Gambling model with Unknown Win Probabilities. **(5)**

(c) Derive the Expectation and Variance of Present value gain by using Mean Variance analysis of Risk Neutral Priced call option. **(15)**

(OR)

(d) Derive the pricing Exotic options by simulation. **(15)**

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