LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2016**

MT 5406 – COMBINATORICS

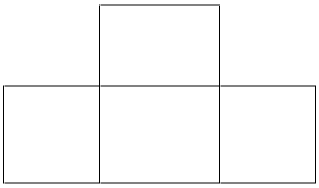
Date: 09-11-2016 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

SECTION-A

ANSWER ALL THE QUESTIONS (10 x 2=20)

1. What is called an n-letter word or word of length n?
2. State exclusion principle.
3. Find the sequence of ordinary generating functions $2x^2(1-x)^{-1}$ and $(3+x)^3$.
4. Define exponential generating functions?
5. State multinomial theorem.
6. Find the coefficient of $x_1^2x_3x_4^3x_5^4$ in $(x_1+x_2+x_3+x_4+x_5)^{10}$.
7. Find the rook polynomial for the chess board c given below



8. Define Euler's function?
9. Define G-equivalence between two sets?
10. Define circular words of length n?

SECTION-B

ANSWER ANY FIVE QUESTIONS (5 x 8=40)

11. There are 30 females, 35 males in a junior class while there are 25 females and 20 males in a senior class. In how many ways can a committee e of 10 be chosen, so that there are exactly 5 females and 3 juniors in the committee?
12. Formulate a table for S_n^m for 1≤m≤51≤n≤5, using sterling formula of first kind.
13. Derive Pascal's identity using the concept of generating functions?
14. Derive the formula for the sum of n natural numbers using recurrence formula.
15. Determine the coefficient of x^{27} in (i) $(x^4+x^5+x^6+......)^5$;

    (ii) $(x^4+2x^5+3x^6+......)^5$.
16. Define permanent of a matrix and find the permanent of $B=\begin{bmatrix} 2 & 2 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 2 & 1 & 2 & 1 \end{bmatrix}$.
17. Find the rook polynomial for 4X4 chess board by the use of expansion formula.
18. State and prove Polya's enumeration theorem?

SECTION-C

ANSWER ANY TWO QUESTIONS **(2x 20=40)**

19. a) Prove that the number of distributions of n distinct objects into m distinct boxes with the objects in the each box arranged in a definite order is $[m]^n$.

b) Define stirling numbers of second kind. Formulate a table for s_n^m for $1 \le m,n \le 6$. **(10+10)**

20. a) State and prove Sieves formula.

b) How many integers 1 and 300 are divisible by (i) at least 3, 5, 7; (ii) 3 and 5 but not 7; (iii) 5 but neither by 3 nor 7. **(12+8)**

21. State and solve ménage problem.

22. a) Let G be symmetric group of a square with vertices labeled as 1,2,3&4 clockwise find the elements of G and the cycle index of G.

b) State and prove Burnside Frobenius theorem. **(10+10)**

****