LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2016**

MT 5505 – REAL ANALYSIS

Date: 01-11-2016 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART –A

ANSWER **ALL** THE QUESTIONS: (10 × 2 = 20 marks)

1. State Principle of mathematical induction.
2. State the triangle inequality.
3. Define (a) Open ball and (b) Closed ball.
4. Define (a) Cauchy sequence and (b) Complete metric space.
5. Define Homeomorphism.
6. Give an example of a continuous function which is not uniformly continuous.
7. Define local minimum and local maximum of a function at a point.
8. Give an example of a continuous function which is not differentiable.
9. Define monotonic functions.
10. State the linear property of Riemann-Stieltjes Integral.

PART - B

ANSWER ANY **FIVE** QUESTIONS: (5 × 8 = 40 marks)

11. Write the field axioms of the set of all real numbers.
12. State and prove Minkowski's inequality.
13. Define convergent sequence and prove that a sequence cannot converge to two distinct limits.
14. Define continuous function and prove that composition of two continuous functions is a continuous function.
15. State and prove Bolzano's theorem.
16. State and prove intermediate value theorem for derivatives.
17. If f is monotonic on $[a,b]$, then prove that the set of discontinuities of f is countable.
18. Give an example of a function which is neither continuous nor of bounded variation.

PART - C

ANSWER ANY **TWO** QUESTIONS: (2 × 20 = 40 marks)

19. (a) State and prove unique factorization theorem. (10)
    (b) State and prove Cauchy Schwarz inequality. (10)
20. (a) Prove that a subset E of a metric space (X,d) is closed in X if, and only if, it contains all its adherent points. (10)
    (b) Prove that the Euclidean space $\mathbb{R}^k$ is complete. (10)
21. (a) If (X,d) and (Y,d) are metric spaces, X is compact and $f: X \rightarrow Y$ is continuous on X, then prove that f is uniformly continuous on X. (11)
    (b) Define uniformly continuous and give an example. (04)
    (c) Discuss the discontinuity of the function $f(x) = \begin{cases} \frac{\sin x}{x} & \text{when } x \neq 0 \\ 2 & \text{when } x = 0 \end{cases}$ (05)
22. (a) State and prove Rolle 's Theorem. (10)
    (b) State and prove integration by parts rule. (10)

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