LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – **NOVEMBER 2016**

MT 6606 – COMPLEX ANALYSIS

(FROM 12-BATCH)

Date: 14-11-2016 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART – A

Answer ALL questions. Each questions carries 2 marks (10 x 2 = 20 marks)

1. Give an example of a function that has infinite limit at ∞.
2. Find an analytic function whose real part is $x^2 - y^2$.
3. Define Cross Ratio.
4. Find the fixed points of the transformation $w = \frac{1}{z}$.
5. Evaluate $\int_C \frac{dz}{z-a}$, where C is a circle with centre 'a' and radius r units.
6. State Cauchy's inequality.
7. Find the singular points of $f(z) = \frac{1}{\sin z}$.
8. Obtain the Taylor's series expansion of $f(z) = \cos z$ about $z = \frac{\pi}{2}$.
9. Find the residue of $f(z) = \frac{z-2}{z(z-1)}$, about z = 0.
10. Write down the formula for evaluating the residue at a pole of order 1.

PART – B

Answer any FIVE questions: Each question carries 8 marks (5 x 8 = 40 marks)

11. Derive the Cauchy – Rieimann equations in polar form.
12. If $f(z)$ is regular and harmonic function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\log\left[|f'(z)|\right] = 0.$
13. Find the linear fractional transformation which maps the points -1, 0, 1 onto the points –i, 1, i, .
14. Show that $\int_{C_1} \bar{z}\, dz = -\pi i,$ where C_1 is the upper half of the circle with centre at origin and radius 1 unit.
15. State and prove Lioville's theorem.
16. Obtain the Laurent's series expansion of $f(z) = \frac{z}{(z-1)(z-3)}$, valid in $0 < |z-1| < 2.$
17. State and prove Argument theorem.
18. Using residue theorem, prove that $\int_C \frac{5z-2}{z(z-1)} dz = 10\pi i, \text{ where C is } |z| = 2.$

PART – C

Answer any TWO questions. Each questions carries 20 marks (2 x 20 = 40 marks)

19. a) Show that the function $f(z) = e^{-z^{-4}}, z \neq 0$
$f(0) = o,$ is not analytic at z = 0,
although Cauchy – Riemann equations are satisfied at the point.

b) If $f(z) = u + iv$ and u-v=e$^x(\cos y - \sin y)$ find f(z) interms of z. (12+8)

20. a) Prove that the cross ratios are preserved under bilinear transformation.

b) If $f(a) = \int_C \frac{3z^2 + 7z + 1}{(z-a)} dz,$ where C is the positively oriented circle $|z| = 2,$ find the values of $f'(1-i), f''(1-i).$ (8+12)

21. a) State and prove Laurent's theorem.

b) State and prove Cauchy's residue theorem. (12+8)

22. a) Using Contour integration, show that $\int_0^{2\pi} \frac{d\theta}{1 + a\sin\theta} = \frac{2\pi}{\sqrt{1-a^2}}, (-1 < a < 1).$ (10+10)

b) Prove that all the roots of $z^7 - 5z^3 + 12 = 0$ lie between the circle $|z| = 1$ and $|z| = 2.$

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