LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc.DEGREE EXAMINATION –**MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2017**

16UMT3MC02– VECTOR ANALYSIS AND ORDINARY DIFF. EQUATIONS

Date: 07-11-2017 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

PART – A

Answer ALL Questions: $10 \times 2 = 20$

1. If $\emptyset(x,y,z) = x^2 + 3y^2 + 2z^2$find $\nabla\emptyset$at the point (2,0,1).
2. If $\vec{F} = xz^3\vec{\imath} - 2xyz\vec{\jmath} + xz\vec{k}$,find$curl\vec{F}$ at (1,2,0).
3. Show that the vector field f, where $f = (y + y^2 + z^2)\vec{\imath} + (x + z + 2xy)\vec{\jmath} + (y + 2zx)\vec{k}$, is conservative.
4. If $\vec{F} = 3xy\vec{\imath} - y^2\vec{\jmath}$, evaluate $\int_C \vec{F}\cdot d\vec{r}$, where C is the curve on the xy plane $y = 2x^2$ from (0,0)to (1,2).
5. State Green's theorem in plane.
6. Find the maximum value of the directional derivative of$\emptyset = 2x^2 + 3y^2 + 5z^2$ at$(1,1,-4)$.
7. Solve:$\frac{dy}{dx} + \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}} = 0$.
8. Solve:$y = 2px + y^2p^3$.
9. Solve:$(D^2 - D - 2)y = 0$.
10. Find the particular integral of $(D^2 - 6D + 9)y = e^{3x}$.

PART – B

Answer ANY FIVE Questions: $5 \times 8 = 40$

11. Prove that for any vector $\vec{A}$, $\nabla \times \left(\nabla \times \vec{A}\right) = \nabla\left(\nabla.\vec{A}\right) - \nabla^2.\vec{A}$
12. Find the directional derivative of $xyz - xy^2z^3$ at the point $(1,2,-1)$, in the direction of the vector $\hat{\imath} - \hat{\jmath} - 3\hat{k}$.

13. Evaluate $\iint_S \vec{F}.\vec{n}ds$ where $\vec{F} = z\vec{i} + x\vec{j} - 3y^2z\vec{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$.

14. Find the value of the integral $\int_C A.\overrightarrow{dr}$, where $A = yz\vec{i} + zx\vec{j} - xy\vec{k}$ if C is the curve whose parametric equations are $x = t, y = t^2, z = t^3$ drawn from $O(0,0,0)$ to $Q(2,4,8)$.

15. Find by Green's theorem the value of $\int_C (x^2 ydx + ydy)$ along the closed curve C formed by $y^2 = x$ and $y = x$ between (0,0) and (1,1).

16. Solve: $xdy - ydx = \sqrt{x^2 + y^2}dx$.

17. Solve: $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$.

18. Solve: $(D^2 + 3D + 2)y = e^{2x} + x^2$.

PART – C

Answer ANY TWO Questions: $\mathbf{2 \times 20 = 40}$

19. a) If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $|\vec{r}| = r$, prove that (i) $\nabla r = \frac{\vec{r}}{r}$ (ii) $\nabla(\log r) = \frac{\vec{r}}{r^2}$

b) Evaluate $\iiint_V \nabla.\vec{F}dV$ if $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ and if V is the volume of the region enclosed by the cube $0 \leq x, y, z \leq 1$.

20. Verify Gauss's theorem for $\vec{A} = x\vec{i} + y\vec{j} + z\vec{k}$ taken over the region bounded by the planes $x = 0, x = a, y = 0, y = a, z = 0, z = a$.

21. a) Solve: $\frac{dy}{dx} - ytanx = \frac{sinxcos^2x}{y^2}$.

b) Solve: $p^2 + 2ypcotx - y^2 = 0$.

22. a) Solve: $(D^2 + 1)y = x^2cosx$.

b) Solve: $\frac{d^2y}{dx^2} + a^2y = secax$, using variation of parameters.

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