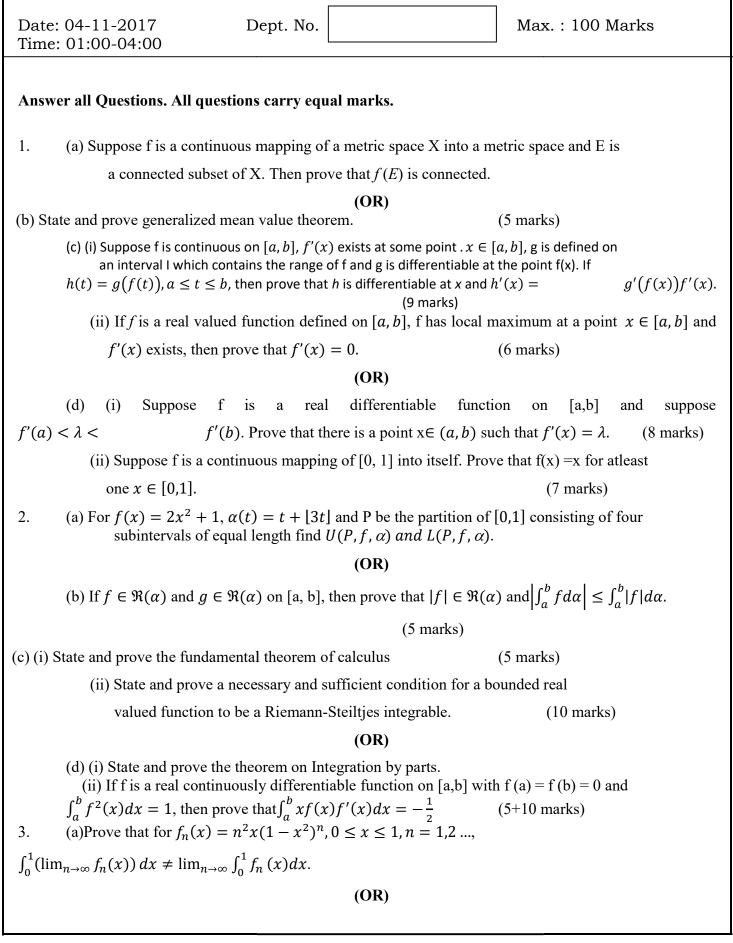
LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc.DEGREE EXAMINATION – MATHEMATICS

FIRSTSEMESTER – NOVEMBER 2017

17PMT1MC02- REAL ANALYSIS



(b)State and prove the Cauchy criterion for uniform convergence of sequence of functions

(5 marks)

(c) If $\{f_n\}$ is a sequence of continuous functions on a set E and if $f_n \to f$ uniformly on E, then prove that f is continuous on E. (15marks)

(**OR**)

(d)State and prove the Stone-Weierstrass theorem. (15 marks)

4. (a) Let= {φ₀, φ₁, φ₂, ... } be orthormal on I and assume that f ∈ L²(I). Define two sequences of functions {s_n} and {t_n} on I as follows: s_n(x) = ∑[∞]_{k=0} c_kφ_k(x), t_n(x) = ∑[∞]_{k=0} b_kφ_k(x) where c_k = (f, φ_k(x) for k = 0, 1, 2... and b₀, b₁, b₂ ... are arbitrary complex numbers. Then for each n, prove that ||f - s_n|| ≤ ||f - t_n||.

(**OR**)

(b)State and prove the Riesz-Fischer theorem. (5 marks)

(c) State and prove the Riemann-Lebesgue lemma and use the lemma to prove the following: For $f \in L(-\infty, +\infty)$, $\lim_{\alpha \to \infty} \int_{-\infty}^{\infty} f(t) \frac{1-\cos\alpha t}{t} dt = \int_{0}^{\infty} \frac{f(t)-f(-t)}{t} dt$. (15 marks)

(OR)

(d) (i) If g is of bounded variation on $[0, \delta]$, then prove that $\lim_{\alpha \to \infty} \frac{2}{\pi} \int_0^{\delta} g(t) \frac{\sin \alpha t}{t} dt = g(0+).$

(ii) Suppose that g(0+) exists and for some $\delta > 0$, the Lebesgue integral $\int_0^{\delta} \frac{g(t)-g(0+)}{t} dt$ exists.

Prove that $\lim_{\alpha \to \infty} \frac{2}{\pi} \int_0^{\delta} g(t) \frac{\sin \alpha t}{t} dt = g(0+).$ (9+6 marks)

5. (a) If Ω is the set of all invertible linear operators on \mathbb{R}^n and for $A \in \Omega, B \in L(\mathbb{R}^n)$,

if $||B - A|| ||A^{-1}|| < 1$, then prove that $B \in \Omega$.

(**OR**)

(b) State and prove the fixed point theorem.

(c) State and prove the inverse function theorem.

(**OR**)

(d) State and prove the implicit function theorem.

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(5 marks)

(15 marks)