LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc.DEGREE EXAMINATION – **MATHEMATICS**

FIRSTSEMESTER – NOVEMBER 2017

17PMT1MC03- ORDINARY DIFFERENTIAL EQUATIONS

Date: 08-11-2017 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

Answer all questions. Each question carries 20 marks.

1. (a) Prove that $uL(v) - vL(u) = a_0(t)\frac{d}{dt}W[u,v] + a_1(t)W[u,v]$, where u, v are twice differentiable functions and a_0, a_1 are continuous on *I*. (5)

(OR)

(b) Determine whether the given sets of functions are linearly dependent or independent. (i) $\{e^x\ ,\ e^{-x}\}$, (ii) $\{1+x,\ 1-x, 1-3x\}$. (5)

(c) Explain the method of variation of parameters for the second order equation. (15)

(OR)

(d) Derive the various solutions of the second order linear homogenous equation with constant coefficients. (15)

2. (a) Let $P_l(x)$ be the Legendre's polynomial. Prove that $P_l(x) = \frac{1}{2^l l!}\frac{d^l}{dx^l}(x^2-1)^l$. (5)

(OR)

(b) Obtain the indicial roots of the equation $x(1-x)\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} - y = 0$. (5)

(c) Obtain the generating function for the Legendre polynomials. (15)

(OR)

(d) Prove that $\int_{-1}^{1} P_l(x)\, P_m(x) dx = \begin{cases} 0 & ; \quad l \neq m \\ 2/(2l+1) & ; \quad l = m \end{cases}$. (15)

3. (a) Prove that $e^{\frac{1}{2}x\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} t^n J_n(x)$. (5)

(OR)

(b) Prove that $J_n'(x) = \frac{n}{x}J_n(x) - J_{n+1}(x)$. (5)

(c) Solve:$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2-n^2)y = 0, n \geq 0$. (15)

(OR)

(d) Let $J_n(x)$ be the Bessel's function of first kind or order *n* and $Y_n(x) = \frac{\cos n\pi\, J_n(x) - J_{-n}(x)}{\sin n\pi}$. Prove that the two independent solutions of the Bessel's equation may be taken to be $J_n(x)$ and $Y_n(x)$for all values of *n*. (15)

4. (a) State and prove Grownwall inequality. (5)

(OR)

(b) For distinct parameters λ and μ, let x and y be the corresponding solutions of the Sturm-Liouville problem such that $[pW(x,y)]_A^B = 0$. Prove that $\int_A^B r(s)x(s)y(s)ds = 0$. (5)

(c) Prove that $x(t)$ is a solution of $L[x(t)] + f(t) = 0, a \leq t \leq b$ if and only if $x(t) = \int_a^b G(t,s)f(s)\,ds$ where $G(t,s)$ is the Green's function. (15)

(OR)

(d) State and prove Picard's theorem for initial value problem. (15)

5. (a) Prove that the null solution of the equation $x' = A(t)x$ is asymptotically stable if and only if $|\phi(t)| \to 0$ as $t \to \infty$. (5)

(OR)

(b) Define an autonomous system and state the stability behaviours of the system. (5)

(c) Explain Lyapunov's direct method. (15)

(OR)

(d) State and prove the fundamental theorems on the stability behaviours of the non-autonomous systems. (15)

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