LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRSTSEMESTER – NOVEMBER 2017

17/16PMT1MC05- PROBABILITY THEORY AND STOCHASTIC PROCESS

Date: 14-11-2017 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

Answer **ALL** questions:

1. (a) A random variable X is distributed at random between the values 0 and 1 so that its probability density function is $f(x) = kx^2(1-x^3)$, where k is a constant. Find the value of k. Using this value of k, find its mean and variance. **(5)**

(OR)

(b) A random variable has the following probability function:

Values of X, x	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	$3k$	k^2	$2k^2$	$7k^2+k$

(i) Find k, (ii) Evaluate $P(X<6)$(iii) If $P(X \leq a) > \frac{1}{2}$, find the minimum value of a. **(5)**

(c) Let (X, Y) be a two dimensional random variable uniformly distributed over the triangular region bounded by $y = 0, x = 3$ and $y = \frac{4}{3}x$. Obtain the correlation coefficient between X and Y. **(15)**

(OR)

(d) Two ideal dice are thrown. Let X_1 be the score of the first die and X_2 the score on the second die. Let Y denote the maximum of $X_1$and X_2. (i) Write down the joint distribution of Y and X_1, and (ii) Find the mean and variance of Y and covariance (Y, X_1). **(15)**

2. (a)State and prove Chebychev's inequality. **(5)**

(OR)

(b) Two unbiased dice are thrown. If X is the sum of the numbers showing up, prove that $P(|X-7| \geq 3) \leq \frac{35}{54}$. Compare this with the actual probability. **(5)**

(c) State and prove weak law of large numbers. **(15)**

(OR)

(d) State and prove two Borel-Cantelli Lemmas. **(15)**

3. (a)Obtain the minimum variance bound (MVB) estimator for μ in normal population $N(\mu, \sigma^2)$, where σ^2 is known. **(5)**

(OR)

(b) State and prove invariance property of Consistent Estimators. **(5)**

(c) If T_1 and $T_2$are minimum variance unbiased (M. V. U.) estimators for $\gamma(\theta)$, then prove that $T_1 = T_2$, almost surely. **(15)**

(OR)

(d) (i) If a sufficient estimator exists then prove that it is a function of Maximum Likelihood Estimator.

(ii) Find the maximum likelihood estimate for the parameter λ of a Poisson distribution on the basis of a sample of size n. Also find its variance. **(7+8)**

4. (a) Explain Critical Region and two types of Errors. **(5)**

(OR)

(b) Let p be the probability that a coin will fall with up head in a single toss in order to test $H_0: p = \frac{1}{2}$against$H_1: p = \frac{3}{4}$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of type I error and power of the test. **(5)**

(c) Let $X \sim N(\mu, 4)$, μ unknown. To test $H_0: \mu = -1$ against $H_1: \mu = 1$, based on a sample of size 10 from its population, we use the critical region: $x_1 + 2x_2 + \cdots + 10x_{10} \geq 0$. What is its size?. What is the power of the test?

(15)

(OR)

(d) State and prove Neyman-Pearson Lemma. **(15)**

5.(a)Explain the four different classes of stochastic processes. **(5)**

(OR)

(b) Explain the Birth and Death processes. **(5)**

(c) Prove that a homogeneous Markov chain $\{X_n\}$ satisfies the relation $p_{ij}^{(n+m)} = \sum_k p_{ik}^{(m)} p_{kj}^{(n)}$ for every $n, m \geq 0$ provided we define $p_{ij}^{(0)} = \delta_{ij}$, where $\delta_{ij} = \begin{cases} 1 & if\ i = j \\ 0 & if\ i \neq j \end{cases}$. **(15)**

(OR)

(d) If the initial vector $\boldsymbol{P}^{(0)}$ is given, then prove that the n −step transition probabilities are $\boldsymbol{P}^{(n)} = \boldsymbol{P}^{(0)} \boldsymbol{P}^{\boldsymbol{n}}$, $n = 1,2, \ldots.$ **(15)**

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