## LOYOLA COLLEGE (AUTONOMOUS), CHENNAI - 600034

## B.Sc.DEGREE EXAMINATION -MATHEMATICS

FIRST SEMESTER - NOVEMBER 2017
17/16UMT1MC01- ALGEBRA AND CALCULUS - I

Date: 06-11-2017
Time: 01:00-04:00
Dept. No. $\square$ Max. : 100 Marks

## PART A

## ANSWER ALL THE QUESTIONS

( $10 * 2=20$ marks $)$

1. Find the $\mathrm{n}^{\text {th }}$ derivative of $y=(a x+b)^{m}$.
2. Find the sub tangent to the curve $y^{2}=4 a x$.
3. How to examine a function $f(x, y)$ is minimum?
4. State the Lagrange's method.
5. Write down Cartesian formula for the radius of curvature .
6. Prove that the (p-r) equation of the cardioid $r=a(1-\cos \theta)$ is $p^{2}=\frac{r^{3}}{2 a}$.
7. Form a rational cubic equation which shall have for roots $1,3-\sqrt{-2}$.
8. Define reciprocal equation.
9. Define evolute.
10. Find the upper limit of the positive roots of the equation $x^{4}-2 x^{3}-13 x^{2}+38 x-24=0$.

## PART B

## ANSWER ANY FIVE QUESTIONS

11. Find the $\mathrm{n}^{\text {th }}$ differential coefficient of $\cos x \cdot \cos 2 x \cdot \cos 3 x$.
12. Find the slope of the tangent with the initial line for the cardioid $r=a(1-\cos \theta)$ at $\theta=\frac{\pi}{6}$.
13. If $u=a^{3} x^{2}+b^{3} y^{2}+c^{3} z^{2}$ where $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$, find the minimum value of $u$.
14. Show that the equation $x^{5}-6 x^{2}-4 x+5=0$ cannot have more than one negative root, using Descartes' rule.
15. Find the asymptotes of the equation $y^{3}-6 x y^{2}+11 x^{2} y-6 x^{3}+x+y=0$.
16. Solve the equation $\mathrm{x}^{4}-5 \mathrm{x}^{3}+4 \mathrm{x}^{2}+8 \mathrm{x}-8=0$ of which one root is $1-\sqrt{5}$.
17. Show that the roots of the equation $x^{3}+p x^{2}+q x+r=0$ are in arithmetical progression if $2 p^{3}-9 p q+27 r=0$.
18. Diminish the roots of the equation $x^{4}+8 x^{3}+2 x^{2}-6 x-9=0$ by 3 .

## PART C

## ANSWER ANY TWO QUESTIONS

$$
(2 * 20=40 \text { marks })
$$

19. a) If $y=\sin \left(m \sin ^{-1} x\right)$, prove that $\left(1-x^{2}\right) y_{2}-x y_{1}+m^{2} y=0$ and $\left(1-x^{2}\right) y_{n+2}-(2 n+1) x y_{n+1}+\left(m^{2}-\right.$ $\left.n^{2}\right) y_{n}=0$.
b) Find the angle of intersection of the cardioids $r=a(1+\cos \theta)$ and $r=b(1-\cos \theta)$.
(10+10)
20. Find the maximum and minimum value of the function $x^{3} y^{2}(6-x-y)$.
21. a) Solve the reciprocal equation $6 x^{6}-35 x^{5}+56 x^{4}-56 x^{2}+35 x-6=0$.
b) Find the sum of the fifth power of the roots of the equation
$x^{6}+5 x^{5}+x^{4}+2 x^{3}-3 x^{2}+8 x+6=0$.
22. a) Solve the equation $81 x^{3}-18 x^{2}-36 x+8=0$ whose roots are in harmonic progression.
b) The equation $x^{3}-2 x^{2}-3 x-4=0$ has a root between 3 and 4 . Calculate it to two places of decimal using Horners's method.
