LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2017**

MT 1503– ANALYTICAL GEOMETRY OF 2D,TRIG. & MATRICES

Date: 04-11-2017 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART – A

ANSWER **ALL** THE QUESTIONS: **(10 × 2 = 20 marks)**

1. Write the expansion of $\tan 4\theta$ in powers of $\tan\theta$.
2. If $x = \cos\theta + i\sin\theta$, then find $x^n + \frac{1}{x^n}$ and $x^n - \frac{1}{x^n}$.
3. Prove that $\cosh^2 x - \sinh^2 x = 1$.
4. Find the general value of logarithm of $x + iy$.
5. Define a singular matrix and a symmetric matrix with an example.
6. Find A^4 using characteristic equation of A when $A = \begin{bmatrix} -1 & 2 \\ -1 & 4 \end{bmatrix}$.
7. Show that the perpendicular tangents to a parabola intersect on the directrix.
8. State any two properties of conjugate diameters.
9. Define rectangular hyperbola.
10. Define polar coordinates.

PART - B

ANSWER ANY **FIVE** QUESTIONS: **(5 × 8 = 40 marks)**

11. Prove that $\cos^5\theta\sin^3\theta = \frac{-1}{128}(\sin 8\theta + 2\sin 6\theta - 2\sin 4\theta - 6\sin 2\theta)$
12. Evaluate $\lim_{x\to\frac{\pi}{2}}\left(\frac{\sin x+\cos 2x}{\cos^2 x}\right)$
13. If $\cos(x + iy) = \cos\theta + i\sin\theta$, then prove that $\cos 2x + \cosh 2y = 2$.
14. Deduce the expansion of $\tan^{-1}x$ in powers of x from the expansion of $\log(a + ib)$.
15. Verify the following matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ satisfies its characteristic equation.
16. Find the locus of the mid-points of chords of a parabola subtending a right angle at the vertex of a parabola.
17. (a) Find the locus of the middle points of a series of parallel chords of an ellipse. **(5+3)**

    (b) When will the tangents at the extremities of a chord intersect on the diameter bisecting the chord.
18. The asymptotes of a hyperbola are parallel to $2x + 3y = 0$ and $3x - 2y = 0$. Its centre is at (1,2) and it passes through the point (5,3). Find its equation and its conjugate.

<u>**PART – C**</u>

ANSWER ANY **TWO** QUESTIONS: **($2 \times 20 = 40$ marks)**

19. (a) Express $\cos 8\theta$ in terms of $\sin \theta$. **(10)**

    (b) Expand $\sin^7\theta$ in a series of sines of multiples of θ. **(10)**

20. (a) If $\cos \alpha \cosh \beta = \cos \emptyset$, $\sin \alpha \sinh \beta = \sin \emptyset$, then prove that $\sin \emptyset = \pm \sin^2\alpha = \pm \sinh^2\beta$**(10)**

    (b) Separate into real and imaginary parts of $\tan^{-1}(x + iy)$ **(10)**

21. Diagonalize the matrix $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$.**(20)**

22. (a) Show that the locus of the intersection of tangents to $y^2 = 4ax$ which intercept a constant length $d$on the directrix is $(y^2 - 4ax)(x + a)^2 = d^2x^2$. **(12)**

    (b) Trace the curve $\frac{12}{r} = 4 + \sqrt{3}\cos\theta + \sin\theta$**.** **(08)**

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