LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc.DEGREE EXAMINATION – **MATHEMATICS**

FIRSTSEMESTER – NOVEMBER 2017

MT 1819- PROBABILITY THEORY AND STOCHASTIC PROCESS

Date: 10-11-2017 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

Answer ALL Questions:

1. (a) A random variable X has the following probability function:

x	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

(i) Find k, (ii) Evaluate $P(X<6), P(X\geq 6)$, and $P(0<X<5)$ (iii) If $P(X\leq a)>\frac{1}{2}$, find the minimum value of a.

OR

(b) The joint probability distribution of two random variables X and Y is given by: $P(X=0,Y=1)=\frac{1}{3}$, $P(X=1,Y=-1)=\frac{1}{3}$, and $P(X=1,Y=1)=\frac{1}{3}$. Find (i) Marginal distribution of X and Y, and (ii) the conditional probability distribution of X given Y=1. (5)

(c) If the random variables X_1 and X_2 are independent and follow Chi-square distribution with p. d. f., show that $\sqrt{n}(X_1-X_2)/2\sqrt{X_1X_2}$ is distributed as student's t with n.d.f., independently of X_1+X_2.

(d) A random sample of 10 boys had the following I.Q's : 70, 120, 110, 101, 85, 83, 95, 98, 107, 100. Do these data support the assumption of a population mean I.Q. of 100? Find the reasonable range in which most of the mean I.Q values of sample of 10 boys lie. (8+7)

OR

(e) Derive the constants of F- distribution.

(f) Let *X* be a continuous random variable with p.d.f given by

$$F(x)=\begin{cases} kx, & 0\leq x<1 \\ k, & 1\leq x<2 \\ -kx+3x, & 2\leq x<3 \\ 0, & elsewhere \end{cases}$$

(i)Determine the constant *k*, (ii) Determine *F(x),* the c.d.f and (iii) If $x_1, x_{2,} x_3$ are three independent observation from *X*, what is the probability that exactly one of these three numbers is larger than 1.5? (5 + 10)

2. (a)Does there exist a variate X for which $P[\mu_x-2\sigma\leq X\leq \mu_x+2\sigma]=0.6$?

OR

(b) State and prove weak law of large number. (5)

(c) State and prove the converse of Borel- Cantelli lemma.

(d) Use Chebyshev's inequality to determine how many times a fair coin must be tossed in order that the probability will be at least 0.90 that the ratio of the observed number of heads to the number of tosses will lie between 0.4 and 0.6. (10 + 5)

OR

(e) If the variables are uniformly bounded, then prove that the condition, $\lim_{n\to\infty}\frac{B_n}{n^2}=0$, is necessary as well as sufficient for WLLN to hold.

State and prove Khinchin's theorem. (5+10)

3. (a)Estimate α and β in the case of Pearson's Type III distribution by the method of moments: $f(x;\ \alpha,\beta)=\frac{\beta^{\alpha}}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}, 0\le x<\infty$.

OR

(b) Let $x_1,x_2,\dots,x_n$ be a random sample from a uniform population on $[0,\theta]$. Find a sufficient estimator for θ. (5)

(c) If T_1 and T_2 be two unbiased estimators of $\gamma(\theta)$ with variances ${\sigma_1}^2,{\sigma_2}^2$ and correlation ρ, what is the best unbiased linear combination of T_1 and T_2 and what is the variance of such a combination? (8)

(d) State and prove the invariance property of consistent estimator. (7)

OR

(e) Find the most likelihood estimator of the parameters α and λ(λ being large) of the distribution $f(x;\alpha,\lambda)=\frac{1}{\Gamma(\lambda)}\left(\frac{\lambda}{\alpha}\right)^{\lambda}e^{\frac{-\lambda x}{\alpha}}x^{\lambda-1}, 0\le x\le\infty,\lambda>0,$ where $\frac{\partial}{\partial\lambda}\log\Gamma(\lambda)=\log\lambda-\frac{1}{2\lambda}$ and $\frac{\partial^2}{\partial\lambda^2}\log\Gamma(\lambda)=\frac{1}{\lambda}+\frac{1}{2\lambda^2}$. (8)

(f) A random sample (X_1,X_2,X_3,X_4,X_5) of size 5 is drawn from a normal population with unknown mean μ. Consider the following estimators to estimate μ:

(i) $t_1=\frac{X_1+X_2+X_3+X_4+X_5}{5}$, (ii) $t_2=\frac{X_1+X_2}{2}+X_3$, (iii) $t_3=\frac{2X_1+X_2+\lambda X_3}{3}$

where λ is such that t_3 is an unbiased estimator of μ. Find λ. Are t_1 and t_2 unbiased? State giving reasons, the estimator which is best among t_1, t_2 and t_3. (7)

4. (a) Derive a relation between two successive ordered observations of a probability density function with continuous random samples $Z_1,Z_2,\dots,Z_n$.

OR

(b) Write a short note on sign test. (5)

(c) Write a brief note on Wald-Wolfowitz run test. (8)

(d) If $x\ge 1$, is the critical region for testing $H_0:\theta=2$ against the alternative $\theta=1$, on the basis of the single observation from the population $f(x,\theta)=\theta e^{-\theta x}, 0\le x<\infty$, obtain the values of type I and type II errors. (7)

OR

(e) Given the frequency function $f(x,\theta)=\begin{cases}\frac{1}{\theta} & 0<x<\infty,\theta>0\\ 0 & elsewhere\end{cases}$ and what you are testing the null hypothesis $H_0:\theta=1$ against $H_1:\theta=2$, by means of a single observed value of x, what would be the sizes of the type I and type II errors, if you choose the interval (i) $0.5\le x$, (ii) $1\le x\le 1.5$ as the critical region? Also obtain the power function of the test.

(f) State and prove Neyman Pearson Lemma. (8 + 7)

5. (a) Write a short note on classification of stochastic process.

OR

(b)A continuous random variable X has aprobability density function $f(x)=3x^2, 0\le x\le 1$. Find a and b such that (i) $P(X\le a)=P(X>a)$, and (ii) $P(X>b)=0.05$. (5)

(c) Prove that a homogeneous Markov chain $\{X_n\}$ satisfies the relation $P_{ij}^{(n+m)}=\sum_k P_{ik}^{(n)}P_{kj}^{(m)}$. (8)

(d) Let P be the transition probability matrix of a finite Markov chain with elements $p_{ij}(i,j=0,1,2,\dots,k-1)$. Then prove that n-step transition probabilities $p_{ij}^{(n)}$ are obtained as the elements of the matrix P^n. (7)

OR

(e) Briefly explain a time dependent general birth and death process in stochastic process. (20)