THIRDSEMESTER - NOVEMBER 2017
MT 3206- APPLIED MATHEMATICS
Dept. No. $\square$ Max. : 100 Marks
Time: 01:00-04:00

## SECTION A

ANSWER ALL QUESTIONS

1. If the revenue function is $R^{\prime}(x)=12-8 x+x^{2}$, determine the total revenue and demand function.
2. Integrate $3 x^{2}+2 x+6$ with respect to $x$.
3. If $\vec{F}=t^{3} \vec{\imath}+t^{2} \vec{\jmath}+(3 t+1) \vec{k}$ find $\frac{d^{2} \vec{F}}{d t^{2}}$.
4. Show that $\vec{F}=z \vec{\imath}+x \vec{\jmath}+y \vec{k}$ is solenoidal.
5. Find the differential equation of all spheres of radius 5 having their centres in the $x y$-plane.
6. Determine the order oft $t^{2} \frac{d^{2} s}{d t^{2}}-s t \frac{d s}{d t}=s$.
7. Define linear programming.
8. Find $L[\sin 2 t]$.
9. Find the value of $L^{-1}\left(\frac{1}{s+4}\right)$.
10. Define Spearman's rank correlation coefficient.

## SECTION B

ANSWER ANY FIVE QUESTIONS
11. The quantity demanded and price under pure competition are determined by the demand and supply function $y=36-x^{2}$ and $y=6+\frac{x^{2}}{4}$ respectively. Determine producer surplus.
12. Find the values of $a, b, c$ so that the vector $\vec{F}=(x+2 y+a z) \vec{\imath}+(b x-3 y-z) \vec{\jmath}+(4 x+c y+$ $2 z) \vec{k}$ is irrotational.
13. Evaluate $\iint e^{\frac{y}{x}} d x d y$ over the region bounded by the straight lines $y=x, y=0$ and $x=1$.
14. Evaluate $\int_{0}^{\infty} e^{-2 t} \sin 3 t d t$.
15. Find the Laplace transform of the following function $f(t)= \begin{cases}t, & 0<t<1 \\ 0, & 1<t<2\end{cases}$
16. Find $L^{-1}\left[\frac{1}{(s+1)(s+3)}\right]$.
17. Determine the maximum and minimum of a function $f(x, y, z)=x y+10 x-x^{2}-y^{2}-z^{2}$.
18. From the following data calculate the coefficient of correlation.

| X | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Y | 10 | 20 | 30 | 50 | 40 |

## SECTION C

## ANSWER ANY TWO QUESTIONS

19. (a) The quantity sold and the corresponding price, under monopoly is determined by the demand function $y=16-x^{2}$ and the marginal cost function $y^{\prime}=6+x$ in such a way as to maximize the profit. Determine the corresponding consumer surplus.
(b) Evaluate $\int_{0}^{a} \int_{0}^{\sqrt{a^{2}-x^{2}}} d x d y$.
20. (a) Evaluate $\iint_{s} \vec{F} \cdot \vec{n} d s$ where $\vec{F}=z \vec{\imath}+x \vec{\jmath}-y^{2} z \vec{k}$ and $S$ is the surface of the cylinder $x^{2}+y^{2}=1$ Included in the first octant between the planes $z=0$ and $z=2$.
(b) Show that the vector $3 x^{2} y \vec{\imath}-4 x y^{2} \vec{\jmath}+2 x y z \vec{k}$ is solenoidal.
21. (a) Solve $\frac{d^{2} y}{d t^{2}}+4 \frac{d y}{d t}-5 y=5$ given that $y(0)=2, y^{\prime}(0)=2$ when $t=0$.
(b) In a culture of east, the amount $A$ of active yeast grows at a rate proportional to the amount present. If the original amount $A_{0}$ doubles in 2 hours, how long does it for the original amount to trip?
22. Calculate the mean, standard deviation, coefficient of variation and variance for the following data:

| Roll. No | 5 | 15 | 25 | 35 | 45 | 55 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Marks | 10 | 20 | 30 | 50 | 40 | 30 |

