



LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc.DEGREE EXAMINATION – MATHEMATICS

FIFTHSEMESTER – NOVEMBER 2017

MT 5408- GRAPH THEORY

Date: 15-11-2017
Time: 09:00-12:00

Dept. No.

Max. : 100 Marks

PART – A

Answer ALL the questions

(10 x 2 = 20)

1. Define a complete bipartite graph with example.
2. Prove that $\delta \leq \frac{2q}{p} \leq \Delta$.
3. Define degree of any vertex of any graph with an example.
4. When a $v_n - v_0$ walk is said to be closed?
5. Define distance between any two vertices of a graph.
6. Define an Eulerian graph and give an example.
7. Prove that every Hamiltonian graph is 2-connected.
8. Define a spanning tree with examples.
9. Define an eccentricity of a vertex v in a connected graph G .
10. Define chromatic number with an example.

PART – B

Answer any FIVE questions

(5 x 8 = 40)

11. Show that in a group of two or more people, there are always two with exactly same number of friends inside the room.
12. If Let G_1 be a (p_1, q_1) graph and G_2 be a (p_2, q_2) graph then prove that
 - (i) $G_1 + G_2$ is a $(p_1 + p_2, q_1 + q_2 + p_1p_2)$ graph.
 - (ii) $G_1 \times G_2$ is a $(p_1p_2, q_1p_2 + q_2p_1)$ graph.
13. Define self complementary graph and prove that any self complementary graph has $4n$ or $4n+1$ vertices.
14. Let v be the vertex of a graph G , then prove that the following statement are equivalent.
 - (i) v is a cut vertex of G .
 - (ii) There exist partition of $G - \{v\}$ into U and W such that for each $u \in U$ and $w \in W$, the edge x is on every $u - w$ path.
 - (iii) There exist two vertices u, w distinct from v such that v is on every $u - w$ path
15. If G is a graph with $p \geq 3$ vertices and $\delta \geq \frac{p}{2}$, then prove that G is Hamiltonian.

16. State and prove five colour theorem for a planar graph.
17. If G is connected planar graph having V, E and F as the set of vertices, edges and faces respectively then, prove that $|V| - |E| + |F| = 2$.
18. Prove that K_5 and $K_{3,3}$ are non planar graphs.

PART – C

Answer any TWO questions

(2 x 20 = 40)

- 19.(a) Define connected graph and prove that a graph G with p vertices and $\delta \geq \frac{p-1}{2}$ is connected.
- (b) The maximum number of edges among all p vertex graphs with no triangles is $\left\lfloor \frac{p^2}{4} \right\rfloor$.
(6+14)
20. (a) If G is not connected then prove that \bar{G} is connected.
- (b) Prove that a graph G with atleast two vertices is bipartite iff all its cycle are of even length.
(5 +15)
- 21.(a) Prove that the following statements are equivalent for a connected graph G
- (i) G is Eulerian.
 - (ii) Every point of G has even degree.
 - (iii) The set of edges of G can be partitioned into cycles.
- (b) If G is a graph in which the degree of every vertex is atleast two then prove that G contains a cycle.
(12 + 8)
- 22.(a) Let G be a (p, q) graph then prove that the following statements are equivalent
- (i) G is a tree.
 - (ii) Every two points of G are connected by a unique path.
 - (iii) G is connected and $p = q + 1$.
 - (iv) G is acyclic and $p = q + 1$.
- (b) If G is a connected (p, q) planar graph with no triangle and $p \geq 3$ then prove that $q \leq 2p - 4$.
(12 + 8)
