LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2017**

MT 5505– REAL ANALYSIS

Date: 10-11-2017 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

SECTION – A

Answer ALL questions: (10 × 2 = 20)

1. Give an example of a subset of real numbers which is not order complete.
2. Differentiate countable and uncountable sets.
3. Define interior point of a set in a metric space.
4. Give an example of a countable collection of open sets whose intersection is not open in a metric space.
5. When do you say that a function has a removable discontinuity at a point *c*?
6. Give an example of a continuous function which is not uniformly continuous.
7. Define (i) strictly increasing function and (ii) strictly decreasing function.
8. Define local minimum and local maximum of a function at a point.
9. Give an example of a function which is not Riemann integrable.
10. Define telescoping series.

SECTION – B

Answer any FIVE questions: (5 × 8 = 40)

11. If n ε N and n is not the square of any integer, show that $\sqrt{n}$ is irrational.
12. Show that subset of a countable set is countable.
13. Let Y be subspace of a metric space (X,d) . Show that a subset A of Y is open in Y if and only if A = Y ∩ G for some open set G in X.
14. Prove that Euclidean space R^k is complete.
15. State and prove Rolle's Theorem.
16. State and prove Bolzano theorem.
17. State and prove intermediate value theorem for derivatives.
18. Show that a function f of bounded variation on [a,b] is bounded on [a,b].

SECTION – C

Answer any TWO questions: **(2 × 20 =40)**

19 (a) Show that countable union of countable sets are countable.

(b) If $e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots$, show that e is irrational. **(10+10)**

20 (a) Show that arbitrary union of open sets is open and finite intersection of open sets is open.

(b) Show that every convergent sequence is Cauchy but not conversely. **(10+10)**

21 (a) If f and g are continuous functions at $x_0 \varepsilon X$ show that fg is also continuous at x_0 and if $f(x_0) \neq 0$ show that 1/f is continuous at x_0.

(b) Define uniformly continuous function. Let *X* be a compact metric space, *Y* be a metric space and $f: X \to Y$ be continuous. Show that *f* is uniformly continuous.

(10+10)

22(a) State and prove Taylor's theorem.

(b) If f, g $\varepsilon R(\alpha)$ on [*a, b*],show that for constants λ, μ , λf + μg ε $R(\alpha)$ on [*a, b*] and $\int_a^b (\lambda f + \mu g) d\alpha = \lambda \int_a^b f d\alpha + \mu \int_a^b g\, d\alpha$. **(10+10)**

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