## LOYOLA COLLEGE (AUTONOMOUS), CHENNAI - 600034

B.Sc. DEGREE EXAMINATION - MATHEMATICS

FIFTH SEMESTER - NOVEMBER 2017
MT 5507 - OPERATIONS RESEARCH

Date: 03-11-2017
Time: 09:00-12:00
Dept. No. $\square$ Max. : 100 Marks

## PART-A

Answer all the questions

1. Define Surplus variables.
2. What is an unbounded solution?
3. What is a transportation problem?
4. Give the Mathematical formulation of an assignment problem.
5. Define Payoff matrix.
6. Define saddle point.
7. Write any two differences between CPM and PERT.
8. Define critical path in a network.
9. What is Economic order quantity?
10. Define Lead Time.

## PART-B

Answer any FIVE questions
11. Use the graphical method to solve the LPP. Minimize $Z=-x_{1}+2 x_{2}$ subject to constraints (i) $-x_{1}+3 x_{2} \leq 10$ (ii) $x_{1}+x_{2} \leq 6$ (iii) $x_{1}-x_{2} \leq 2$ and $x_{1}, x_{2} \geq 0$
12. Find the minimal weight spanning tree for the following interconnection network.

13. Determine an initial basic feasible solution to the following transportation problem by using (a) North West Corner Method (b) Least Cost Method.

Source

|  | $D_{1}$ | $D_{2}$ | $D_{3}$ | $D_{4}$ | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $A$ | 11 | 13 | 17 | 14 | 250 |
| $B$ | 16 | 18 | 14 | 10 | 300 |
| $C$ | 21 | 24 | 13 | 10 | 400 |
| Demand | 200 | 225 | 275 | 250 |  |

14. A department of a company has five employees with five jobs to be performed. The time (in hours) that each man takes to perform each job is given in the effectiveness matrix. How should the jobs be allocated, one per employee, so as to minimize the total man-hours?

Employees

| Jobs | A | I | II | III | IV | V |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 10 | 5 | 13 | 15 | 16 |
|  | $B$ | 3 | 9 | 18 | 13 | 6 |
|  | C | 10 | 7 | 2 | 2 | 2 |
|  | D | 7 | 11 | 9 | 7 | 12 |
|  | E | 7 | 9 | 10 | 4 | 12 |

15. Solve the following game after reducing it to $\mathrm{a} 2 \times 2$ game

Player B

| Player $A$ | $B_{1}$ | $B_{2}$ | $B_{3}$ |
| :---: | :---: | :---: | :---: |
| $A_{1}$ | 1 | 7 | 2 |
| $A_{2}$ | 6 | 2 | 7 |
| $A_{3}$ | 5 | 1 | 6 |

16. Solve the following game by using graphical method and find the value of the game.

Player B

| Player $A$ | $B_{1}$ | $B_{2}$ | $B_{3}$ | $B_{4}$ |
| :---: | :---: | :---: | :---: | :---: |
| $A_{1}$ | 2 | 2 | 3 | -2 |
| $A_{2}$ | 4 | 3 | 2 | 6 |

17. An assembly is to be made from two parts $X$ and $Y$. Both parts must be turned on a lathe, $Y$ must be polished whereas X need not be polished. The sequence of activities, together with their predecessors, is given below.

| Activity | Description | Predecessor Activity |
| :---: | :---: | :---: |
| $A$ | Open work order | - |
| $B$ | Get material for $X$ | $A$ |
| $C$ | Get material for $Y$ | $A$ |
| $D$ | Turn $X$ on lathe | $B$ |
| $E$ | Turn $Y$ on lathe | $B, C$ |
| $F$ | Polish $Y$ | $E$ |
| $G$ | Assemble $X$ and $Y$ | $D, F$ |
| $H$ | Pack | $G$ |

Draw a network diagram of activities for the project.
18. A company that operates for 50 weeks in a year is concerned about its stocks of copper cable. This costs Rs 240 a meter and there is a demand for 8,000 meters a week. Each replenishments costs Rs 1,050 for administration and Rs 1,650 for delivery, while holding costs are estimated at 25 percent of value held a year. Assuming no shortages are allowed, what is the optimal inventory policy for the company? How would this analysis differ if the company wanted to maximize its profits rather than minimize cost? What is the gross profit if the company sells the cable for Rs 360 a meter?

## PART-C

19. Use the Simplex method to solve the following LP Problem.

Mazimize $Z=3 x_{1}+5 x_{2}+4 x_{3}$ subject to constraints (i) $2 x_{1}+3 x_{2} \leq 8$, (ii) $2 x_{2}+5 x_{3} \leq 10$,
(iii) $3 x_{1}+2 x_{2}+4 x_{3} \leq 15$ and $x_{1}, x_{2}, x_{3} \geq 0$.
20. (a) A company has factories at $F_{1}, F_{2}$ and $F_{3}$ that supply products to warehouses at $W_{1}, W_{2}$ and $W_{3}$.The weekly capacities of the factories are 200,160 and 90 units, respectively. The weekly warehouse requirements are 180,120 and150 units respectively. The unit shipping costs (in rupees) are as follows:

Warehouse

|  | $W_{1}$ | $W_{2}$ | $W_{3}$ | Supply |
| :---: | :---: | :---: | :---: | :---: |
| Factory | $F_{1}$ | 16 | 20 | 12 |
|  | $F_{2}$ | 14 | 8 | 18 |
|  | $F_{3}$ | 26 | 24 | 16 |
|  | 180 | 120 | 150 | 450 |

Determine the optimal distribution for this company in order to minimize its total shipping cost.
(b) A travelling salesman has to visit five cities. He wishes to start from a particular city, visit each city once and then return to his starting point. The travelling cost(in' 000 Rs ) of each city from a particular city is given below:

| From City |  | To City |  |  | D | $E$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | A | B | C |  |  |
|  | A | $\infty$ | 2 | 5 | 7 | 1 |
|  | $B$ | 6 | $\infty$ | 3 | 8 | 2 |
|  | C | 8 | 7 | $\infty$ | 4 | 7 |
|  | D | 12 | 4 | 6 | $\infty$ | 5 |
|  | E | 1 | 3 | 2 | 8 | $\infty$ |

What should be the sequence of visit of the salesman so that the cost is minimum?
21. (a) Solve the game whose payoff matrix is given below:

Player $B$

| Player $A$ | $B_{1}$ | $B_{2}$ | $B_{3}$ | $B_{4}$ |
| :---: | :---: | :---: | :---: | :---: |
| $A_{1}$ | 3 | 2 | 4 | 0 |
| $A_{2}$ | 3 | 4 | 2 | 4 |
| $A_{3}$ | 4 | 2 | 4 | 0 |
| $A_{4}$ | 0 | 4 | 0 | 8 |

(b) A small project is composed of 7 activities whose time estimates are listed in the table below. Activities are identified by their beginning ( $i$ ) and ending ( $j$ ) node numbers.

| Activity | Estimated Duration (weeks) |  |  |
| :---: | :---: | :---: | :---: |
| $(i-j)$ | Optimistic | Most Likely | Pessimistic |
| $1-2$ | 1 | 1 | 7 |
| $1-3$ | 1 | 4 | 7 |
| $1-4$ | 2 | 2 | 8 |
| $2-5$ | 1 | 1 | 1 |
| $3-5$ | 2 | 5 | 14 |
| $4-6$ | 2 | 5 | 8 |
| $5-6$ | 3 | 6 | 15 |

(i) Draw the network of the activities in the project
(ii) Find the expected duration and variance for each activity.
(iii) What are the expected project length and critical path?
(iv) Calculate the standard deviation of the project length.
22. (a) The production department of a company requires $3,600 \mathrm{~kg}$ of raw materials for manufacturing a particular item per year. It has been estimated that the cost of placing an order is Rs 36 and the cost of carrying inventory is 25 percent of the investment in the inventories .The price is Rs 10 per kg. Help the purchase manager to determine an ordering policy for raw material.
(b) The annual demand of a product is 10000 units. Each unit costs Rs100 if the orders are placed in quantities below 200 units. For orders of 200 or above, however, the price is Rs 95.The annual inventory holding costs is 10 percent of the value of the item and the ordering cost is Rs 5 per order. Find the economic lot size.

