LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION –**MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2017**

MT 5509– ALGEBRAIC STRUCTURE - II

Date: 06-11-2017 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART –A

ANSWER ALL THE QUESTIONS: (10 X 2 = 20)

1. Define a vector space V over a field F.
2. If V is a vector space over a field F, show that $(-a)v = a(-v) = -(av)$ for $a \in F, v \in V$.
3. Define a basis of a vector space.
4. Define homomorphism of a vector space into itself.
5. Define inner product space.
6. Prove that the product of two invertible linear transformations on V is itself an invertible linear transformations on V.
7. Show that the matrix A = $\begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}$ is orthogonal.
8. If A and B are Hermitian, show that AB – BA is Skew- Hermitian.
9. Define symmetric matrix.
10. If T^*T = I, then show that T is unitary.

PART - B

ANSWER ANY FIVE QUESTIONS: (5 X 8 = 40)

11. Prove that the intersection of two subspaces of a vector space V is a subspace of V.
12. If S and T are subsets of a vector space V over F, then prove that
    (i) S is a subspace of V if and only if L(S) $\subseteq$ S.
    (ii) S $\subseteq$ T implies that L(S) $\subseteq$ L(T).
    (iii) L(L(S)) = L (S).
13. If V is a vector space of finite dimension, and is the direct sum of its subspaces U and W, then prove that $dimV = dimU + dimW$.
14. State and prove Schwarz inequality.
15. If $T \in A(V)$ is singular if and only if there exists an element $v \neq 0$ in V such that $T(v) = 0$.
16. Let $V = R^3$, and let $T \in A(V)$ be defined by $T(a_1, a_2, a_3) = (3a_1 + a_3, -2a_1 + a_2, -a_1 + 2a_2 + 4a_3)$. What is the matrix of T relative to the basis $v_1 = (1,0,1), v_2 = (-1,2,1), v_3 = (2,1,1)$?

17. Solve the system of linear equations

$x_1 + 2x_2 + 2x_3 = 5, \quad x_1 - 3x_2 + 2x_3 = -5, \; 2x_1 - x_2 + x_3 = -3.$

over the rational field by working only with the augmented matrix of the system.

18. If $< T(v), T(v) > = < v, v >$ for all v in V, then prove that T is unitary.

PART – C

ANSWER ANY TWO QUESTIONS: **(2 X 20 = 40)**

19. (i) The vector space V over F is a direct sum of two of its subspaces $W_1 \, and \, W_2$ if and only if $V = W_1 \oplus W_2$ and $W_1 \cap W_2 = (0)$. **(10+10)**

(ii) Express the vector (1, -2, 5) as a linear combination of the vectors (1,1,1) , (1,2,3) ,and (2,-1,1) in R^3,where R is the field of real numbers.

20. If V is a vector space of finite dimension and W is a subspace of V, then prove that $dim\frac{V}{W} = dimV - dimW$.

21. Prove that every finite – dimensional inner product space V has an orthonormal set as a basis.

22. If $T \in A(V)$, then $T^* \in A(V)$. Moreover,

(i) $(S+T)^* = S^* + T^*$

(ii) $(ST)^* = T^* S^*$

(iii) $(\lambda T)^* = \bar{\lambda} T^*$

(iv) $(T^*)^* = T$.

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