

# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034



## B.Sc. DEGREE EXAMINATION – MATHEMATICS

FIFTH SEMESTER – NOVEMBER 2017

### MT 5510 – STATICS

Date: 08-11-2017

Dept. No.

Max. : 100 Marks

Time: 09:00-12:00

#### PART – A

Answer ALL questions.

(10 × 2 = 20)

1. Let  $\mathbf{F}$  be a force acting at a point  $O$ . Find the components of  $\mathbf{F}$  along the direction  $OX$  which makes an angle  $\alpha$  with  $\mathbf{F}$  and  $OY$  which makes an angle  $\beta$  with  $\mathbf{F}$ .
2. State triangular law of forces.
3. Define moment of a force.
4. Define a couple.
5. Give an example of a body where the centre of mass is not necessarily a point of the body.
6. What is meant by centre of gravity of a compound body?
7. Define unstable equilibrium.
8. State equation of virtual work.
9. Define span of a catenary.
10. What is a suspension bridge?

#### PART – B

Answer any FIVE questions

(5 × 8 = 40)

11. Two forces of magnitude  $P$  and  $Q$  ( $P > Q$ ) act on a particle and the angle between the forces is  $\alpha$ . If the magnitudes of the forces are interchanged, show that the resultant turns through the angle  $2 \tan^{-1} \left( \frac{P-Q}{P+Q} \tan \frac{\alpha}{2} \right)$ .
12. State and prove Lami's theorem.
13. A heavy carriage wheel of weight  $W$  and radius  $r$  is to be dragged over an obstacle of height  $h$  by a horizontal force of magnitude  $F$  applied to the centre of the wheel. Show that  $F$  must be greater than  $\frac{W\sqrt{2rh-h^2}}{r-h}$ .
14. Find the centre of gravity of a thin uniform triangular lamina.
15. Find the resultant of two like parallel forces.

16. A uniform rod of length  $2a$  rests in equilibrium against a smooth vertical wall and upon a smooth peg at a distance  $b$  from the wall. If in the position of equilibrium the rod is inclined at an angle  $\theta$  with the wall, show that  $\sin\theta = \left(\frac{b}{a}\right)^{\frac{1}{3}}$ .
17. Derive Cartesian equation of the catenary.
18. A uniform chain of length  $l$  is to be suspended from two points A and B, in the same horizontal line so that either terminal tension is  $n$  times that at the lowest point. Show that the span AB must be  $\frac{l}{\sqrt{n^2-1}} \log(n + \sqrt{n^2-1})$ .

**PART – C**

Answer any TWO questions.

(2 × 20 = 40)

- 19 (a) Discuss the equilibrium of a heavy particle on a smooth inclined plane.  
 (b) State and prove Varignon's theorem of moments. (10+10)
- 20 (a) Two strings AB and AC are knotted at A, where a weight  $W$  is attached. If the weight hangs freely and in the position of equilibrium, with BC horizontal, AB: BC : CA = 2:4:3, show that the tensions in the strings are  $\frac{7W}{2\sqrt{15}}$  and  $\frac{11}{4\sqrt{15}}$ .  
 (b) Two rough particles connected by a light string rest on an inclined plane. If their weights and corresponding coefficients of friction are  $W_1, W_2$  and  $\mu_1, \mu_2$  respectively and  $\mu_1 > \tan\alpha > \mu_2$  where  $\alpha$  is the inclination of the plane with the horizon, prove that  $\tan\alpha = \frac{\mu_1 W_1 + \mu_2 W_2}{W_1 + W_2}$ , if both particles are on the point of moving down the plane. (10+10)
- 21 (a) Find the centre of gravity of the segment of a sphere of radius  $a$  cut off by a plane at distance  $c$  from the centre,  
 (b) A string of length  $l$  hangs between two points not in the same vertical line and the tangents at the end points are inclined at an angle  $\alpha$  and  $\beta$  with the horizontal. Show that the height of one extremity above the other is  $\frac{l \sin \frac{\alpha+\beta}{2}}{\cos \frac{\alpha-\beta}{2}}$ , the two extremities being on the same side of the vertex of the catenary. (10+10)
- 22 (a) A rod lies in equilibrium with its ends on two smooth planes inclined at an angle  $\alpha, \beta$  to the horizontal, the planes intersecting in a horizontal line. Show that the inclination of the rod to the horizontal is  $\tan^{-1} \left( \frac{\sin(\alpha-\beta)}{2 \sin\alpha \sin\beta} \right)$ .  
 (b) State and prove the principle of virtual work for a system of coplanar forces acting on a rigid body. (10+10)

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