LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – MATHEMATICS

FIRST SEMESTER – NOVEMBER 2019

16/17/18UMT1MC02 – ANALYTICAL GEOMETRY OF 2D, TRIG. MATRICES

Date: 01-11-2019   Dept. No.   Max. : 100 Marks
Time: 09:00-12:00

PART- A

Answer ALL the questions $(10 \times 2 = 20)$

1.   Write down the expansion of $\tan\theta$ in ascending powers of θ up to θ^5.

2.   If $x = \cos_n + i\,\sin_n$, then find $x^2 - \dfrac{1}{x^2}$.

3.   Show that $\cosh^2 x - \sinh^2 x = 1$.

4.   Find the general value of $\log_e(x + iy)$.

5.   Write the characteristic equation of a matrix.

6.   State Cayley Hamilton theorem.

7.   Find the poles of the line $2x + y + 5 = 0$ with respect to the ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$.

8.   Find the equation of the ellipse whose vertices are $(2, -2)$, $(2,4)$ and eccentricity is $\dfrac{1}{3}$.

9.   What is the equation of straight line and a circle in polar form.

10.   Find the equation of the hyperbola whose eccentricity is $\sqrt{2}$ and the distance between foci is 16, taking transverse and conjugate axes of the hyperbola as x and y axes respectively.

PART- B

Answer any FIVE questions $(5 \times 8 = 40)$

11.   Express $\dfrac{\sin 6_n}{\sin_n}$ in terms of $\cos\theta$.

12.   Evaluate $\underset{x \to f/2}{Lt} \dfrac{\sin x + \cos 2x}{\cos^2 x}$.

13.   Prove that $\tanh^{-1} x = \dfrac{1}{2}\log_e\left(\dfrac{1+x}{1-x}\right)$.

14.   If $\tan(x + iy) = u + iv$, prove that $\dfrac{u}{v} = \dfrac{\sin 2x}{\sinh 2y}$.

15. Find the value of $A^3 + 2A^2 - A$ if $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$.

16. Find the eigenvalues and eigen vectors of $A = \begin{pmatrix} -2 & 2 \\ 2 & 1 \end{pmatrix}$.

17. Find the locus of the poles of chords of a parabola subtending a right angle at vertex.

18. Find the asymptotes of the hyperbola $3x^2 + 10xy + 8y^2 + 14x + 22y + 7 = 0$.

PART- C

Answer any TWO questions $(2 \times 20 = 40)$

19. (a) Expand $\sin^3\theta\cos^5\theta$ in a series of multiples of θ.

   (b) Evaluate $\underset{\text{''} \to 0}{Lt} \dfrac{\tan\text{''} + \sec\text{''} - 1}{\tan\text{''} - \sec\text{''} + 1}$. **(10+10)**

20. (a) If $\cos(x + iy) = \cos\text{''} + i\sin\text{''}$, prove that $\cos 2x + \cosh 2y = 2$.

   (b) Separate the real and imaginary parts of $\tan^{-1}(x + iy)$. **(10+10)**

21. Diagonalise the matrix $\begin{pmatrix} -1 & 0 & 2 \\ 0 & 1 & 2 \\ 2 & 2 & 0 \end{pmatrix}$. **(20)**

22. (a) Trace the curve $\dfrac{2}{r} = 1 + \cos\text{''} + \sin\text{''}$.

   (b) Show that the conjugate lines through a focus of an ellipse are at right angles. **(10+10)**

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