LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2019**

16/17/18UMT3MC01 – INTEGRAL TRANSFORMS AND PARTIAL DIFF. EQUATIONS

Date: 29-10-2019 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

SECTION – A

Answer ALL questions. **(10 ò 2 = 20)**

1. What is $L(f''(t))$?
2. Find $L(sin^2 2t)$.
3. Find $L^{-1}\left(\frac{1}{(s-3)^2}\right)$.
4. Find $L^{-1}\left(\frac{1}{s^2+k^2}\right)$.
5. Define Fourier transform for a function $f(x)$.
6. Prove that $F\{e^{iax}f(x)\} = F(s+a)$ where $F(s) = F\{f(x)\}$.
7. Define Fourier sine transform.
8. Show that $F_c\{f(ax)\} = \frac{1}{a}F_c\left(\frac{s}{a}\right)$.
9. Form a partial differential equation by eliminating the arbitrary function from $z = f(x^2 + y^2)$.
10. Solve $x + y\frac{\partial z}{\partial x} = 0$.

SECTION – B

Answer any FIVE questions **(5 × 8 = 40)**

11. Find the Laplace transform of the function $f(t) = \begin{cases} \sin t, & 0 < t < \pi \\ 0, & t > \pi \end{cases}$.
12. Find (i) L($t\, sin^2 t$) (ii) Find L($t^2 cos4t$). **(4 + 4)**
13. Find $L^{-1}\left(\frac{s^2}{(s^2+4)(s^2+9)}\right)$.
14. Show that F{$f(ax)$} = $\frac{1}{|a|}F\left(\frac{s}{a}\right)$ for $a \neq 0$.

15. Prove that $F\left\{\frac{d^n}{dx^n} f(x)\right\} = (-is)^n F(s)$, provided $f(x), f'(x), f''(x) \dots f^{n-1}(x) \to 0$ as $x \to \pm\infty$.

16. Show that $F_c\{f''(x)\} = -\sqrt{\frac{2}{\pi}} f'(0) - s^2 F_c(s)$ if $f(x) \to 0, f'(x) \to 0$ as $x \to \infty$.

17. Solve $\sqrt{p} + \sqrt{q} = 1$.

18. Solve $p^2 + q^2 = x + y$.

SECTION – C

Answer any TWO questions. **(2 ò 20 = 40)**

19. Find (a) $L^{-1}\left(\frac{2s^2+10\,s}{(s+1)(s^2-2s+5)}\right)$ (b) $L^{-1}\left(\frac{s-1}{2s^2+s+6}\right)$. **(10 + 10)**

20. Using Laplace transform, solve $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 4e^{-t}$ given that $y = 0$, $\frac{dy}{dt} = 0$ when $t = 0$. **(20)**

21. (a) State and prove convolution theorem.

(b) State and prove Parseval's identity. **(10 + 10)**

22. (a) Find the complete solution of $p(1 + q^2) = q(z - 1)$.

(b) Solve $(y - z)p + (z - x)q = x - y$. **(8 + 12)**

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