LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2019**

18PMT3MC01 – TOPOLOGY

Date: 29-10-2019 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

Answer all the questions. Each question carries 20 marks.

I a)1)Construct Cantor's set and state Cantor's intersection theorem

OR

a)2) Define the following in a topological space: (i) interior point and (ii) boundary point. How do they help in understanding the concept of topological spaces**.** **(5)**

b)1) Let X and Y be metric spaces and f a mapping of X into Y. Then prove that if f is continuous then $f^{-1}(G)$ is open whenever G is open in Y.

b)2) If $\{A_n\}$ is a sequence of nowhere dense sets in a complete metric space X then prove that there exists a point in X which is not in any of the A_n's. **(5+10)**

OR

c)1) If a convergent sequence in a metric space has infinitely many points then prove that its limit is a limit point of the set of points of the sequence.

c)2) Let X be a metric space and let Y be a complete metric space, and let A be a dense subspace of X. If f is uniformly continuous mapping of A into Y then prove that f can be extended uniquely to a uniformly continuous mapping g of X into Y. **(5+10)**

II a)1) What are the other popular names for topology? Justify.

OR

a)2) Prove that every separable metric space is second countable. **(5)**

b)1) State and prove Lindelof's theorem

b)2) State and prove Tychonoff's theorem. **(7+8)**

OR

c)1) State Heine Borel theorem.

c)2) Prove that a topological space is compact if every class of subbasic closed sets with the finite intersection property has non-empty intersection. **(5+10)**

III a)1) State Bolzano-Weierstrass property and define sequentially compactness. How do these concepts help us in understanding the concept of compactness.

OR

a)2) State Lebesgue covering lemma. **(5)**

b) State and prove Ascoli's theorem.

OR

c)Consider the following statements:

(i) X is compact (ii) X is sequentially compact and (iii) X has the Bolzano-Weierstrass property.

Prove that both the statements (i) and (ii) imply the third statement. **(15)**

IV a)1)) Prove that any continuous image of a connected space is connected.

OR

a)2) Prove that the spaces R^n and C^n are connected. **(5)**

b)1) Prove that the range of continuous real function defined on a connected space is an interval.

b)2) Prove that a subspace of a real line R is connected if and only if it is an interval. In particular show that R is connected. **(5+10)**

OR

c)1) Prove that a topological space X is disconnected iff there exists a continuous mapping of X onto the discrete two- point space {0,1}.

c)2) Let X be a compact Hausdorff space. Then prove that X is totally disconnected if and only if it has an open base whose sets are also closed. **(7+8)**

V a)1) State Real and Complex Stone Weierstrass theorems.

OR

a)2) State the two lemma required to prove extended Stone Weierstrauss theorems. **(5)**

b) State and prove Weierstrass approximation theorem.

OR

c) Define locally compact Hausdorff spaces. How can it be made into a compact Hausdorff space. Also, prove that X_∞ is Hausdorff and compact. **(15)**

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