LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2019**

18/17/16UMT3MC02 – VECTOR ANALYSIS AND ORDINARY DIFF. EQUATIONS

Date: 31-10-2019 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

SECTION – A

Answer ALL questions **(10 ò 2 = 20)**

1. If $\phi(x, y, z) = x^2y + y^2x + z^2$ find $\nabla\varphi$ at (1, 1, 1).
2. Show that the vector $\bar{F} = 3y^4z^2\vec{\imath} + 4x^3z^2\,\bar{\jmath} - 3x^2y^2\bar{k}$ is solenoidal.
3. If $\bar{F} = (3x^2 + 6y)\bar{\imath} - 14yz\bar{\jmath} + 20xz^2\bar{k}$, evaluate $\int \bar{F}\,.dr$ along the straight line joining (0,0,0) to (1,0,0).
4. Define conservative field.
5. Find the unit vector normal to the surface $\phi = x^3 - xyz + z^3 - 1$ at the point (1,1,1).
6. State Gauss divergent theorem.
7. Solve $\frac{dy}{dx} = \frac{y+2}{x-1}$.
8. Find the general solution of $y = xp + \frac{\alpha}{p}$.
9. Solve $(D^2 - 6D + 8)y = 0$.
10. Define Legendre Linear equation.

SECTION – B

Answer any FIVE questions **(5 ò 8 = 40)**

11. If $\nabla\varphi = (y^2 - 2xyz^3)\bar{\imath} + (3+2xy - x^2z^3)\,\bar{\jmath} + (6z^3 - 3x^2yz^2)\,\bar{k}$, find ϕ.
12. If $\vec{r} = \vec{r}x\vec{\imath} + y\vec{\jmath} + z\vec{k}$ and $r = |\vec{r}|$, show that $\nabla^2 r^n = n(n+1)r^{n-2}$.
13. Find $\int_C F.dr$ where $\bar{F} = x^2\,\bar{\imath} + y^3\bar{\jmath}$ where C is the portion of the parabola $y = x^2$ in the XY plane from (0,0) to (1,1).
14. Evaluate $\iint_S \bar{F}\,.\bar{n}\,ds$ where $\bar{F} = yz\bar{\imath} + zx\bar{\jmath} + xy\,\bar{k}$ and S is that part of the surface of the sphere $x^2 + y^2 + z^2 = 1$ which lies in the first octant.

15. Using Divergence theorem, evaluate $\iint_S \vec{F} \cdot \hat{n}\, ds$ where $\vec{F} = x^3\,\vec{i} + y^3\,\vec{j} + z^3\,\vec{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$.

16. Solve $y(1 - p^2) = 2px$.

17. Solve $(D^2 - 4D - 5)y = \cos x + e^{-x}$.

18. Solve $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \sin(\log x^2)$.

SECTION – C

Answer any TWO questions **(2 ò 20 = 40)**

19. (a) Prove that $\vec{F} = (y^2\cos x + z^3)\vec{i} + (2y\sin x - 4)\vec{j} + (3xz^2 + 2)\vec{k}$ is irrotational.

(b) If $\vec{F} = 2xz\,\vec{i} - x\,\vec{j} + y^2\,\vec{k}$, then evaluate $\iiint_V \vec{F} \cdot dv$ where V is the region bounded by the surfaces $x = 0$, $y = 0$, $y = 6$, $z = x^2$, $z = 4$. (10+10)

20. Verify Stroke's theorem for the function $\vec{F} = x^2\,\vec{i} + xy\,\vec{j}$ in the region in XY plane bounded by $x=0$; $x=a$; $y=0$ and $y=b$. (20)

21. (a) Solve $\frac{dy}{dx} - y\tan x = \frac{\sin x \cos^2 x}{y^2}$.

(b) Solve $x = 1 - \frac{p}{\sqrt{p^2+1}}$.

(c) Find the general and singular solution of $y = xp + p^2$. (7+7+6)

22. (a) Solve $(D^2 - 2D + 1)y = x \sin x$.

(b) Solve, $\frac{d^2y}{dt^2} + 4y = 4\tan 2x$ using method of variation of parameters. (10+10)

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