LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – PHYSICS

FIRST SEMESTER – NOVEMBER 2019

MT 1100 – MATHEMATICS FOR PHYSICS

Date: 05-11-2019   Dept. No.   Max. : 100 Marks
Time: 09:00-12:00

Part A

Answer ALL questions   **(2 x 10 = 20)**

1. Find the n $^{\text{th}}$ derivative of $y = \sin(ax + b)$.

2. Show that the sub-tangent at any point is double the abscissa for the parabola $y^2 = 4ax$.

3. Write the expansion of $\dfrac{e^x + e^{-x}}{2}$.

4. Determine the characteristic equation of $A = \begin{bmatrix} 7 & 3 \\ 2 & 6 \end{bmatrix}$.

5. Compute $L[\cos 2t]$.

6. Find $L^{-1}\left[\dfrac{1}{(s+a)^2}\right]$.

7. Write $\cos \theta$ in a series of powers of θ.

8. Prove that $\cosh^2 x - \sinh^2 x = 1$.

9. When two unbiased coins are tossed once, what is the probability of getting at least one head?

10. Define Poisson distribution.

Part B

Answer any FIVE questions   **(5 x 8 = 40)**

11. Find the angle of intersection of the curves $r = a\left(1 + \cos \theta\right)$ and $r = b\left(1 - \cos \theta\right)$.

12. Determine the maxima and minima of the function $f(x) = 2x^3 - 3x^2 - 36x + 10$.

13. Find the sum to the infinity of the series $1 + \dfrac{3}{4} + \dfrac{3.5}{4.8} + \dfrac{3.5.7}{4.8.12} + \dfrac{3.5.7.9}{4.8.12.16} + \ldots\ldots\ldots$

14. Verify Cayley – Hamilton theorem for $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$.

15. Find $L[f(t)]$, if $f(t) = \begin{cases} (t-1)^2 & ;\text{when } t > 1 \\ 0 & ;\text{when } t < 1 \end{cases}$.

16. Find $L^{-1}\left[\dfrac{1}{(s-1)(s-2)(s-3)}\right]$.

17. Express $\dfrac{\sin 6\theta}{\sin \theta}$ in terms of $\cos \theta$.

18. Two unbiased dice are thrown. Find the probability that:
    (i) both the dice show the same number,
    (ii) the first die shows 6,
    (iii) the total of the numbers on the dice is 8,
    (iv) the total of the numbers on the dice is any number from 2 to 12, both inclusive.

Part C

Answer any TWO questions   **(2 x 20 = 40)**

19. a) If $y = \sin\left(m\sin^{-1} x\right)$, Prove that $\left(1 - x^2\right)y_2 - xy_1 + m^2 y = 0$.

   b) Solve the differential equation $\dfrac{d^2 y}{dx^2} + 2\dfrac{dy}{dx} + 5y = 4e^{-t}$ given $y(0) = 0$ and $y'(0) = 0$ using Laplace

     transform.

                    (8 + 12)

20. a) Find the sum of the series $1 + \dfrac{1+5}{2!} + \dfrac{1+5+5^2}{3!} + \ldots\ldots\ldots + \infty$.

   b) Show that $\log\sqrt{12} = 1 + \left(\dfrac{1}{2} + \dfrac{1}{3}\right)\dfrac{1}{4} + \left(\dfrac{1}{4} + \dfrac{1}{5}\right)\dfrac{1}{4^2} + \left(\dfrac{1}{6} + \dfrac{1}{7}\right)\dfrac{1}{4^3} + \ldots\ldots$

                    (8 + 12)

21.   a) Expand $\sin^6 \theta$ in series of cosines of multiples of θ.

   b) Express $\cos 8\theta$ in terms of $\sin \theta$.

                    (10 + 10)

22.   a) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

   b) Calculate the mean for the following table giving the age distribution of 542 members.

Age in years	20 – 30	30 – 40	40 – 50	50 - 60	60 – 70	70 – 80	80 - 90
Number of members	3	61	132	153	140	51	2

                    (12 + 8)

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