LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2019**

MT 1502 – ALGEBRA AND CALCULUS - I

Date: 30-10-2019 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART- A

Answer ALL questions: **(10 x 2 =20)**

1. Find the nth derivative of the function e^{ax}, where a is a constant.
2. Find the polar sub-tangent of the curve $r = ae^{\theta \cot \alpha}$.
3. What is the minimum value of the function $f(x) = x^2$ defined on R?
4. Write the steps used in Lagrange's method of undetermined multipliers.
5. What is the radius of curvature of the circle $x^2 + y^2 = 25$?
6. Write the pedal equation of a curve.
7. Form the cubic equation two of whose roots are 1 and $3+\sqrt{-2}$.
8. What is the sum of the roots of the equation $x^3 + 3x^2 + 2x - 5 = 0$.
9. Define evolute of a curve.
10. State Descartes's rule of signs for negative roots of a polynomial function.

PART-B

Answer any FIVE questions: **(5 x8 = 40)**

11. Find the angle of intersection of the cardioids $r = a(1 + \cos\theta)$ and $r = b(1 - \cos\theta)$.
12. Find the nth derivative of $e^x \sin x$.
13. If $y = e^{a \sin^{-1}x}$, prove that $(1 - x^2)y_2 - xy_1 = a^2y$.
14. Find the maximum and minimum values of the function $f(x) = -x^2 - y^2 + x^2y^2$.
15. Find the minimum value of $f(x) = x^2 + y^2 + z^2$ subject to the constraint $x + y + z = 3a$.
16. Find the radius of curvature of the curve $x^4 + y^4 = 2$ at the point (1,1).
17. If the roots of the equation $x^3 + px^2 + qx + r = 0$ are in arithmetic progression, show that $2p^3 - 9pq + 27r = 0$.
18. Show that the function $x^7 - 3x^4 + 2x^3 = 1$ has at least four imaginary roots.

PART-C

Answer any TWO questions: **(2 x 20 = 40)**

19. Prove the evolute of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ is another cycloid.
20. Show that the sum of the eleventh powers of the roots of the equation $x^7 + 5x^4 + 1 = 0$ is zero.
21. (a). Solve the equation $6x^5 + 11x^4 - 33x^3 - 33x^2 + 11x + 6 = 0$.
    (b). Solve the equation $x^4 + 4x^3 + 5x^2 + 2x = 2$ one of whose roots is $-1 + i$. (10+10)
22. Using Horner's method find the real root of the equation $x^3 - 3x + 1 = 0$ correct to three decimal places.

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