LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2019**

MT 3503 – VECTOR ANALYSIS & ORDINARY DIFF. EQUATIONS

Date: 29-10-2019 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART-A

Answer ALL questions: **(10 × 2 = 20)**

1. Prove that $Curl\ (\vec{r}) = 0$, where $\vec{r}$ is the position vector.
2. Find a such that $(3x - 2y + z)\vec{i} + (4x + ay - z)\vec{j} + (x - y + 2z)\vec{k}$ is solenoidal.
3. Define a conservative vector field.
4. If $\vec{F} = y\,\vec{i} - x\vec{j}$, evaluate $\int_C \vec{F}\,.\,d\vec{r}$ from $(0,0)$ to $(1,1)$ along the curve $y = x$.
5. State Green's theorem.
6. State Stoke's theorem.
7. Solve: $4p^2 - 8p + 3 = 0$, where $p = \frac{dy}{dx}$.
8. Write down the Bernoulli's equation.
9. Solve: $(D^2 - 5D + 6)\,y = 0$.
10. Find the particular integral $(D^2 - 3D + 2)\,y = e^x$.

PART - B

Answer any FIVE questions: **(5 × 8 = 40)**

11. Compute the divergence and curl of the vector $\vec{F} = xy^2\,\vec{i} + 2x^2yz\vec{j} - 3yz^2\vec{k}$ at $(1,-1,1)$.
12. Prove that $\nabla \times (\nabla \times \vec{F}) = \nabla(\nabla\,.\,\vec{F}) - \nabla^2\vec{F}$.
13. Evaluate $\iint_S \vec{A}.\hat{n}\,ds$, where $\vec{A} = y\,z\,\vec{i} + 2\,y^2\,\vec{j} + x\,z^2\,\vec{k}$ and S is the surface of the cylinder $x^2 + y^2 = 9$ included in the first octant between $z = 0$ and $z = 2$.
14. Evaluate $\iiint_V \nabla.\vec{F}dV$ where $\vec{F} = x^2\,\vec{i} + y^2\,\vec{j} + z^2\,\vec{k}$ and V is the volume enclosed by the cube $0 \le x, y, z \le 1$.
15. Solve: $p\,(1 + q^2) = q\,(z - 1)$.
16. Find the general solution of $(y + z)p + (z + x)q = x + y$.
17. Solve: $(D^2 + 5D + 6)y = e^x$.
18. Evaluate: $(D^2 + 16)\,y = cos4x$.

PART - C

Answer any TWO questions: **(2 × 20 = 40)**

19. (a) Prove that $\vec{F} = (y^2 \cos x + z^3)\vec{i} + (2y \sin x - 4)\vec{j} + (3xz^2)\vec{k}$ is irrotational and find it's scalar potential.

(b) Find the value of the integral $\int_C \vec{A} \cdot d\vec{r}$ where $\vec{A} = yz\,\vec{i} + zx\,\vec{j} - xy\,\vec{k}$ is the following cases

(i) C is the curve whose parametric equation are $x = t, y = t^2, z = t^3$

Drawn from $(0,0,0)$ to $(2,4,8)$. (ii) C is the curve obtained joining $(0,0,0)$ to $(2,0,0)$ then $(2,0,0)$ to $(2,4,0)$ and then $(2,4,0)$ to $(2,4,8)$. (10+10)

20. Verify divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ taken over the rectangular parallelepiped $0 \le x \le a, 0 \le y \le b, 0 \le z \le c$. (20)

21. Solve: $(5+2x)^2 \frac{d^2y}{dx^2} - 6(5+2x)\frac{dy}{dx} + 8y = 6x$. (20)

22. Solve: (a) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \log x$. (b) $\frac{d^2y}{dx^2} + y = \sec x$. (10+10)

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