

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034



B.Sc. DEGREE EXAMINATION – MATHEMATICS

THIRD SEMESTER – NOVEMBER 2019

MT 3504 – INTEGRAL TRANSFORMS & PARTIAL DIFF. EQUATIONS

Date: 31-10-2019

Dept. No.

Max. : 100 Marks

Time: 01:00-04:00

Part - A

Answer ALL questions:

(10 x 2 = 20)

1. Eliminate the arbitrary constants from $z = ax + by$.
2. Form a partial differential equation by eliminating the arbitrary function from $z = f(x^2 - y^2)$.
3. Find $(\sin 2t)$.
4. Evaluate $L(t^2 + 2t + 3)$.
5. Find $L^{-1}\left(\frac{s-3}{(s-3)^2+4}\right)$.
6. Find $L^{-1}\left(\frac{s}{s^2+a^2}\right)$.
7. Prove that $F\{af(x) + bg(x)\} = aF(f(x)) + bF(g(x))$.
8. Prove that $\{e(x)\} = F(s + a)$.
9. Define Fourier cosine transform.
10. Prove that $\{(x)\} = (s)$.

Part - B

Answer any FIVE questions:

(5 x 8 = 40)

11. Solve $q = xp + p^2$.
12. Solve $p^2 + q^2 = x^2 + y^2$.
13. Determine $L(te^{-t} \sin t)$.
14. Find the Laplace transform of the periodic function $f(t) = \begin{cases} 1, & 0 < t < b \\ -1, & b < t < 2b \end{cases}$.
15. Determine $L^{-1}\left(\frac{1}{s(s-1)(s-2)}\right)$.
16. Find $L^{-1}\left(\frac{s-3}{s^2+4s+13}\right)$.
17. State and prove any two properties of Fourier sine transform.
18. Show that $F(x^n f(x)) = (-i)^n \frac{d^n}{ds^n} F(f(x))$.

Part - C

Answer any TWO questions:

(2 x 20 = 40)

19. (a) Solve $z = px + qy + \sqrt{1 + p^2 + q^2}$.

(b) Solve $(y^2 + z^2)p - xyz = -xz$.

(10+10)

20. Using Laplace transform, solve the equation $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = \sin t$ given that $y = \frac{dy}{dt} = 0$ when $t = 0$.

(20)

21. (a) State and prove complex form of Fourier integral theorem.

(b) Prove the following:

i) $F_c(xf(x)) = \frac{dF_S}{ds}$.

ii) $F_S(xf(x)) = -\frac{dF_C}{ds}$.

(10+10)

22. (a) State and prove Parseval's identity.

(b) State and prove convolution theorem.

(10+10)
