LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2019**

MT 5509 – ALGEBRAIC STRUCTURE - II

Date: 02-11-2019 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

Part A (Answer ALL questions) **(10x2 = 20)**

1. Define a vector space *V* over a field *F*.
2. If *V* is a vector space over a field *F*, show that $(-a)v = a(-v) = -(av)$ for $a \in F$, $v \in V$.
3. Check whether the linear transformation $T : R^2 \to R$ defined by $T(a,b) = ab$ for all $a,b \in R$ is a vector space homomorphism or not.
4. Define Kernel of a homomorphism *T*.
5. Let R^3 be the inner product space over R under standard inner product. Find the norm of $(3,0,4)$.
6. Define eigen value of a linear transformation *T*.
7. When do you say that a matrix is invertible?
8. Define a symmetric matrix.
9. Find the rank of the matrix $A = \begin{pmatrix} 1 & 5 & -7 \\ 2 & 3 & 1 \end{pmatrix}$ over the field of rational numbers.
10. If $T * T = I$, then show that *T* is unitary.

Part B (Answer any FIVE questions) **(5 x 8 = 40)**

11. Prove that the intersection of two subspaces of a vector space *V* is a subspace of *V*.
12. Express the vector (1, -2, 5) as a linear combination of the vectors (1, 1, 1), (1, 2, 3) and (2, -1, 1) in R^3 over the field of real numbers R.
13. If V and W are two *n*-dimensional vector spaces over F. Then prove that any isomorphism *T* of V onto W maps a basis of V onto a basis of W.
14. Let $T \in A(V)$ and $a \in F$. Prove that a is an eigenvalue of T if and only if $aI - T$ is singular.
15. State and prove Schwarz inequality.
16. Let $V = R^3$ and let $T \in A(V)$ be defined by $T\left(a_1, a_2, a_3\right) = \left(3a_1 + a_3, -2a_1 + a_2, -a_1 + 2a_2 + 4a_3\right)$. What is the matrix of T relative to the basis $v_1 = (1,0,1)$, $v_2 = (-1,2,1)$, $v_3 = (2,1,1)$?
17. Show that any square matrix *A* can be expressed uniquely as the sum of a symmetric and a skew-symmetric matrices.

18. If $< T(v), T(v) > \; = \; < v, v >$ for all $v \in V$ an inner product space, show that T is unitary.

Part C (Answer any TWO questions) **(2 x 20 = 40)**

19. (a) Let V be a vector space over a field F and W be a subspace of V. Prove that V/W is a vector space over F.

(b) If S and T are subsets of a vector space V over F, then prove that the following:

(i) S is a subspace of V if and only if $L(S) = S$.

(ii) $S \subseteq T$ implies that $L(S) \subseteq L(T)$.

(12 + 8)

20. If W_1 and W_2 are subspaces of a finite dimensional vector space V, then prove that $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$.

(20)

21. Prove that every finite dimensional inner product space has an orthonormal set as a basis.

(20)

22. (a) If $A, B \in F_n$ and $\alpha \in F$, then prove that the following.

(i) $(\alpha A)^t = \alpha A^t$, (ii) $\left(A^t\right)^t = A$, (iii) $\left(A + B\right)^t = A^t + B^t$, (iv) $\left(AB\right)^t = B^t A^t$.

(b) Show that the system of following linear equations is inconsistent.

$$x_1 + 2x_2 + x_3 = 11, \quad 4x_1 + 6x_2 + 5x_3 = 8, \quad 2x_1 + 2x_2 + 3x_3 = 19.$$

(10 + 10)

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