LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2022**

PMT 3503 – OPERATIONS RESEARCH

Date: 25-11-2022 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

Answer ALL the Questions

I. (a) Discuss the role of sensitivity analysis in industry.

OR

(b) What is an integer programming problem? Explain the different types of it. **(5 Marks)**

(c) Find the Optimum solution of the following LPP.

$Maximize\ Z = 6x_1 + 8x_2$

subject to the constraints

$5x_1 + 10x_2 \le 60$

$4x_1 + 4x_2 \le 40$

(1) Apply sensitivity analysis to find the solution of the given LPP if the right-hand side vector $\begin{bmatrix} 60 \\ 40 \end{bmatrix}$ is changed to $\begin{bmatrix} 50 \\ 30 \end{bmatrix}$.

(2) Also find out how far the new first resource can be decreased.

OR

(d) Solve the following LP problem by branch and bound method.

$Maximize\ Z = 2x + 3y$

subject to the constraints

$6x + 5y \le 25$

$x + 3y \le 10$ where x and y are non-negative integers **(15 Marks)**

II. (a) Explain ABC analysis. What are its advantages and limitations?

OR

(b) Explain the terms balking, reneging, jockeying, collusion, and queue discipline in queueing theory. **(5 Marks)**

(c) (i) Discuss about deviational variables and pre-emptive priority factors in goal programming.

(ii) A manufacturing firm produces two products A and B. Each product must be processed through two departments. Department I has 60 hours of production capacity per week, and department II has 85 hours per week. Each unit of Product A requires 4 hours in department I and 6 hours in department II. Each unit of product B requires 5 hours in department I and 7 hours in department II. Management has set the following priorities.

Priority	**Goal**
P_1 :	Minimize the underachievement of joint total production of 14 units.
P_2 :	Minimize the underachievement of producing 9 units of product B.
P_3 :	Minimize the overachievement of producing 7 units of product A.

Formulate this problem as a GP problem and illustrate with graph. **(3+12 Marks)**

OR

(d) In a hospital, an emergency care unit treats 96 patients per day. Also on average, one patient requires 10 minutes of active attention. Assume that facility can handle only one emergency at a time. It costs Rs.700 per patient as service charge and 10 minutes of service time. Each minute of decrease in average time would cost Rs.70 per patient treated, how much would have to be budgeted by the hospital to decrease the average size of the queue from $1\frac{1}{3}$ to $\frac{1}{2}$ patients. Also find the average time spend in the queue and in the system. **(15 Marks)**

III. (a) Explain the replacement technique. Explain the terms: money value and resale value.Give examples where they are related and differ.

OR

(b) Explain gradual failure and sudden failure with two examples for each. How can you avoid mechanism failure at home? **(5 Marks)**

(c) (i) Explain individual and group replacement policies with example.

(ii) The cost of an equipment is Rs.1,24,000 and its scrap value is only Rs.4000. The life of the equipment is 8 years. The maintenance cost for each year is as given below:

Year	1	2	3	4	5	6	7	8
Maintenance cost in Rs.	2000	4000	7000	10000	16000	22000	32000	48000

When the equipment should be replaced? **(5+10 Marks)**

OR

(d) Machine A costs Rs.9000. Annual operating cost is Rs.200 for the first year and then increases by Rs.2000 every year. Determine the best age at which the machine to be replaced. Machine B costs Rs.10000. Annual operating cost is Rs.400 for the first year and then increases by Rs.800 every year. For both the machines there is no scrap value. The company owner plans to buy 5 machines with a combination of at least 2 from each type. Give the best combination to reduce the expense. Which machine will you prefer if all the 5 machines are of only one type? **(15 Marks)**

IV. (a) Explain dynamic programming problem and state Bellman's principle of optimality.

OR

(b) Mention some of the applications of dynamic programming and any four types of problems. **(5 Marks)**

(c) (i) Mention the salient features of dynamic programming technique. **(5+10 Marks)**

(ii) A group of students decided to travel from city 1 to city 10. The cost of travel from each city to the other city en route, based on relevant factors such as distance, difficulties, mode of available transportation etc. are given below. Find the least cost route from city 1 to city 10 using dynamic programming technique.

city	2	3	4
1	22	30	34

city	5	6	7
2	70	40	56
3	30	27	40
4	42	10	50

city	8	9
5	12	10
6	61	31
7	35	41

city	10
8	48
9	52

OR

(d) A firm has divided its marketing zone into three zones. The firm has 9 salesmen and data of possible profits in thousands of rupees are given in table. Allocate the salesmen to three zones so that the total sales become maximum using dynamic programming technique.

No of salesmen	Zone 1	Zone 2	Zone 3
0	30	35	42
1	45	45	54
2	60	52	60
3	70	64	70
4	79	72	82
5	90	82	95
6	98	93	102
7	105	98	110
8	100	100	110
9	90	100	110

(15 Marks)

V. (a) Determine the values of x, y, z that maximize or minimize the function $f(x, y) = x^2 + 2y^2 + z^2 + xy - 7x - 2z + 14.$

OR

(b) State Kuhn-Tucker conditions to solve quadratic programming problem. **(5 Marks)**

(c) Using Kuhn-Tucker conditions solve the non-linear programming problem:

Minimize $Z = 5x_1^2 - x_2$

subject to $x_1 + x_2 = 8$

$x_1^2 + 3x_2^2 \leq 5$

$x_1 \geq 2$ where $x_1, x_2 \geq 0.$

OR

(d) Determine optimal solution for the function $f = 2x^2 - 24x + 2y^2 - 8y + 2z^2 - 12z + 300$ subject to the constraint $x + y + z = 11$ and check whether it maximizes or minimizes using Lagrangian Multiplier Method. **(15 Marks)**

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