

**LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034**



**B.Sc. DEGREE EXAMINATION – STATISTICS**

**FIRST SEMESTER – NOVEMBER 2022**

**UMT 1303 – MATHEMATICS FOR STATISTICS**

Date: 01-12-2022

Dept. No.

Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

**PART – A**

**Answer ALL the Questions**

**(10 x 2 = 20)**

1. If  $y = 4x^3 - 2x + \frac{3}{x^3}$ , find  $\frac{dy}{dx}$ .
2. Differentiate  $e^t$  with respect to  $\sqrt{t}$ .
3. For what values of  $x$  is  $2x^3 - 9x^2 + 12x + 4$  is a decreasing function?
4. Find the points of inflexion in the curve  $y = x^3 - 9x^2 + 7x - 6$ .
5. If  $V = (x^2 + y^2 + z^2)^{1/2}$ , find  $\frac{\partial V}{\partial x}$ .
6. Find the first order partial differential coefficients of  $u = \cos(7x + 4y)$ .
7. Evaluate:  $\int \left(x + \frac{1}{x} + e^x\right) dx$ .
8. Evaluate:  $\int \frac{dx}{1+9x^2}$ .
9. Find  $\int_1^3 (2x^2 + x - 4) dx$ .
10. State any properties of definite integral.

**PART – B**

**Answer any FIVE Questions**

**(5 x 8 = 40)**

11. (a) Find the differential coefficient of  $e^x \sin x \log x$  with respect to  $x$ .  
(b) Solve  $\frac{d}{dx} \left( \frac{\sqrt{x}}{2x+3} \right)$ .
12. If  $x(1+y)^{1/2} + y(1+x)^{1/2} = 0$ , prove that  $\frac{dy}{dx} = -\frac{1}{(1+x^2)}$ .
13. Estimate the maximum value of  $\frac{\log x}{x}$  for positive values of  $x$ .
14. Determine  $y_n$ , when  $y = \frac{x^2}{(x-1)^2(x+2)}$ .
15. Estimate the maxima and minima points of the function  $2x^3 - 3x^2 - 36x + 10$ .
16. State Euler's theorem and analyse whether it is true for the function  $u = x^3 + y^3 + z^3 + 3xyz$ .
17. Integrate  $\frac{3x-2}{\sqrt{4x^2-4x-5}}$  with respect to  $x$ .
18. Apply the appropriate property of definite integral and calculate  $\int_0^{\pi/2} \log \sin x \, dx$ .

PART – C

Answer any TWO Questions

(2 x 20 = 40)

19. If  $y = \sin(m \sin^{-1} x)$ , then prove that  $(1 - x^2)y_2 - xy_1 + m^2y = 0$  and  $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y = 0$ .

20. Find the maximum and minimum values of the curve  $u = 2(x^2 - y^2) - x^4 + y^4$ .

21. a) Show that for  $x > 0$ , prove that  $x - \frac{1}{2}x^2 < \log(1 + x) < x$ .

b) If  $u = \log\left(\frac{x^2+y^2}{xy}\right)$ , show that  $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ . (12 + 8)

22. a) Evaluate:  $\int \frac{2x+3}{x^2+x+1} dx$ .

b) Evaluate:  $\int \frac{3x+1}{(x-1)^2(x+3)} dx$ . (10 + 10)

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