LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2022**

UMT 3501 – ABSTRACT ALGEBRA

Date: 24-11-2022 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

SECTION - A			
Answer ALL the Questions			
1.	**Answer the following**	**(5 x 1 = 5)**	
a)	Define equivalence relation on a set S.	K1	CO1
b)	Find the order of 2 in $(Z_8, \oplus)$.	K1	CO1
c)	Express the following permutation as a product of disjoint cycles $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 5 & 1 & 3 \end{pmatrix}$	K1	CO1
d)	What is cipher text?	K1	CO1
e)	Define unit in a ring R.	K1	CO1
2.	**Choose the correct answer**	**(5 x 1 = 5)**	
a)	From the following, which is not a group? a) $(z, +)$ b)(N,+) c) (Q,+) d) None	K1	CO1
b)	If $G$is a group of even order, then it has an element $a \neq e$ satisfies (i) $a^2 = e$ (ii) $a^2 \neq e$ (iii) $a^n = e$ (iv) $a^n \neq e$	K1	CO1
c)	An Isomorphism of G onto itself is called (i) Homomorphism (ii) Epimorphism (iii) Automorphism (iv) Endomorphism	K1	CO1
d)	The characteristic of an integral domain D is a) Zero always b) a prime number always c) either zero or a prime number d) None	K1	CO1
e)	The private key in asymmetric key cryptography is kept by	K1	CO1

	a) Sender b) Receiver c) Sender and receiver d) All the connected devices to the network		
3.	**Fill in the blanks**	**(5 x 1 = 5)**	
a)	If a prime number divides the product of certain integers it must divide at least_________ of these integers.	K2	CO1
b)	In a finite group the order of every element is ___________________ .	K2	CO1
c)	Every permutation is ____________ of its transpositions.	K2	CO1
d)	A commutative division ring is called _____________.	K2	CO1
e)	If n objects are distributed over m places, and if n > m , then some place receives at least two objects. This is called _________________.	K2	CO1
4.	**State True or False**	**(5 x 1 = 5)**	
a)	If a and b are relatively prime, then we can find integers m and n such that $ma + nb = 2$.	K2	CO1
b)	Every cyclic group is abelian.	K2	CO1
c)	Any two cyclic groups with the same number of elements are isomorphic.	K2	CO1
d)	If p is a prime number, then the ring of integers $mod\ p$ is not a field.	K2	CO1
e)	RSA is an algorithm used for symmetric key cryptography.	K2	CO1

SECTION - B

Answer any TWO of the following. **(2 x 10 = 20)**

5.	Prove that any positive integer $a > 1$ can factored in a unique way as $a = p_1^{\alpha_1} p_2^{\alpha_2} \ldots p_t^{\alpha_t}$, where $p_1 > p_2 > \cdots p_t$ are prime numbers and where each $\alpha_i > 0$.	K3	CO2
6.	If G is a finite group and H is a subgroup of G, prove that $o(H)$ is a divisor of o(a).	K3	CO2
7.	If G is a group, then $\mathcal{A}(G)$,the set of all automorphisms of G is also a group.	K3	CO2
8.	Prove that every finite integral domain is a field.	K3	CO2

SECTION - C

Answer any TWO of the following. **(2 x 10 = 20)**

9.	Prove that in a Euclidean ring d is the greatest common divisor of any two elements a & b in R and also prove that $d = \lambda a + \mu b$ for some $\lambda, \mu \in$ R.	K4	CO3
10.	If Φ is a homomorphism of G into $\bar{G}$ with kernel k, prove that k is a normal subgroup of G.	K4	CO3
11.	Let R be a commutative ring with unit element whose only ideals are $\{o\}$ and R itself. Prove that R is a field.	K4	CO3
12.	If H and K are two subgroups of G. Prove that $HK$is a subgroup of G if and only if $HK = KH$.	K4	CO3

SECTION - D			
Answer any ONE of the following.			**(1 x 20 = 20)**
13.	Every group is isomorphic to a subgroup of $A(s)$ for some appropriate S.	K5	CO4
14.	State & prove Unique Factorization theorem in Euclidean ring R.	K5	CO4

SECTION - E			
Answer any ONE of the following.			**(1 x 20 = 20)**
15.	(i)If H and K are finite subgroups of G of orders o(H) and o(K) respectively, then prove that o(HK) = $\frac{O(H)\, O(K)}{O(H\cap K)}$ **(15 Marks)** (ii) Which of the following groups are cyclic? If it is cyclic, find all the generators of the group. (a)The group of symmetries of an equilateral triangle (b) The group of symmetries of a rectangle. (c) (Q,+) (d)(6z,+) (e) $(z_4, \oplus)$ **(5 Marks)**	K6	CO5
16.	(i)If $U$is an ideal of R, prove that R/U is a ring and is a homomorphic image of R. (15 marks) (ii)Find encoded message of HELLO WORLD using Julius Ceasar algorithm. **(5 Marks)**	K6	CO5

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