LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2022**

UMT 5502 – LINEAR ALGEBRA

Date: 25-11-2022 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

PART – A

Q. No	Answer ALL the questions (10 x 2 = 20 Marks)
1	If V is a vector space over F, then prove that $(-\alpha)v = -(\alpha v)$ for $\alpha \epsilon F$ and $v \in V$.
2	Define Linearly dependent vector.
3	If V is a vector space over F,$u \in V$ and $\alpha \in F$,Prove that $\|\alpha u\| = \|\alpha\|\|u\|$.
4	Define an orthonormal set.
5	If V is finite dimensional vector space over F and if $T \in A(V)$ is right invertible, then prove that T is invertible.
6	Define a characteristic vector of a linear transformation T.
7	When do you say $S, T \epsilon A(V)$ are similar?
8	Define invariant subspace.
9	Define Hermitian adjoint.
10	If $T \in A(V)$ is unitary then prove that $TT^* = 1$.

PART – B

Answer any FIVE questions **(5 x 8 = 40 Marks)**

11	(a)	Let V be a vector space over F and S be a nonempty subset of V. Then prove that $L(S)$ is a subspace of V.	**(4)**
	(b)	If F is a field of real numbers, verify whether the vectors $(1,2,1), (2,1,0)$ and $(1,-1,2)$ are linearly independent.	**(4)**
12		State and prove triangular inequality in an inner product space.	**(8)**
13		If $\lambda \in F$ is a characteristic root of $T \in A(V)$,then prove that for any polynomial $q(x) \in F(x), q(\lambda)$ is a characteristic root of $q(T)$.	**(8)**
14		Let V be the vector space of polynomials of degree 3 or less over F. In V define T by $(\alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3)T = \alpha_1 + 2\alpha_2 x + 3\alpha_3 x^2$. Compute the matrix of T in the basis (a) $1,\ x,\ x^2,\ x^3$ (b) $1,\ 1+x,\ 1+x^2,\ 1+\ x^3$	**(8)**

15 Prove that the Hermitian linear transformation T is nonnegative if and only if all of its characteristic roots are nonnegative. **(8)**

16 Prove that if V is finite-dimensional over F, then $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not 0. **(8)**

17 If V is the internal direct sum of $U_1, U_2, \dots, U_n$, then prove that V is isomorphic to the external direct sum of $U_1, U_2, \dots, U_n$. **(8)**

18 Prove that if $T \epsilon A(V)$ then $T^* \epsilon A(V)$. Moreover, for all $S, T \epsilon A(V)$ and all $\lambda \epsilon F$, show that **(8)**

(i) $(T^*)^* = T$.

(ii) $(S + T)^* = S^* + T^*$.

(iii) $(\lambda S)^* = \bar{\lambda} S^*$.

(iv) $(ST)^* = T^* S^*$.

PART – C

Answer any TWO question **(2 x 20 = 20 Marks)**

19 If V and W are of dimensions *m* and *n*, respectively over F, then prove that $Hom\ (V, W)$ is of dimension mn over F. **(20)**

20 (a) State and prove Schwarz inequality **(8)**

(b) Let V be a finite dimensional inner product space, then prove that V has an orthonormal set as a basis **(12)**

21 (a) If V is a finite dimensional over F, show that for $S, T \in A(V)$ **(10)**
(i) $r(ST) \le r(T)$ (ii) $r(TS) \le r(T)$ (iii) $r(ST) = r(TS) = r(T)$ for S regular in $A(V)$.

(b) Prove that the linear transformation T on V is unitary if and only if it takes an orthonormal basis of V into an orthonormal basis of V. **(10)**

22 Prove that if $T \epsilon A(V)$ has all its characteristic roots in F, then there is a basis of V in which the matrix of T is triangular. **(20)**

****