LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2023**

PMT1MC02 – REAL ANALYSIS-I

Date: 03-11-2023 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

	SECTION A – K1 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
1	**Answer the following.**
a)	What is a closure of a set? Give one example.
b)	Verify whether the function $f(x) = x^2 + 1$ satisfies mean value theorem in the interval [1,4].
c)	State the necessary and sufficient condition for $f \in \mathcal{R}(\alpha)$.
d)	Define uniform convergence.
e)	Why is the Weierstrass approximation theorem important?
	SECTION A – K2 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
2	**Choose the correct answer.**
a)	Let X be the metric space which is both complete and totally bounded then is said to be …………… (i) scalar (ii) compact (iii) complete (iv) discrete
b)	If f has a derivative at c then it is ………..at c. (i) Continuous (ii) Bounded (iii) closed (iv) Neither or nor continuous
c)	If $f_1(x) \leq f_2(x)$ on $[a, b]$ then……………….. (i) $\int_a^b f_2 d\alpha \leq \int_a^b f_1 d\alpha$ (ii) $\int_a^b f_2 d\alpha = \int_a^b f_1 d\alpha$ (iii) $\int_a^b f_1 d\alpha \leq \int_a^b f_2 d\alpha$ (iv) $\int_a^b f_2 d\alpha = -\int_a^b f_1 d\alpha$
d)	If $\{f_n\}$ is a sequence of continuous function on E and if $f_n \to f$ uniformly then f is ……… on E. (i) Continuous (ii) Discontinuous (iii) closed (iv) Differentiable
e)	Any continuous function defined on a ……………….can be approximated uniformly by a polynomial function. (i) $[a, \infty)$ (ii) (a, b) (iii) $(-\infty, \infty)$ (iv) [a,b]

	SECTION B – K3 (CO2)
	Answer any THREE of the following **(3 x 10 = 30)**
3	Show that a mapping f of a metric space X into a metric space Y is continuous iff $f^{-1}(V)$ is open in X for every open set V in Y.
4	Let f be defined on $[a,b]$. If f has a local maximum at a point $x \in (a,b)$ and if $f'(x)$ exists, then show that $f'(x) = 0$.
5	a) Show that the lower Riemann-Stieltjes integral cannot exceed the upper Riemann-Stieltjes integral. b) Let $f(x) = x$ and $\alpha(x) = x^2$. Does $\int_0^1 f\, d\alpha$ exists? If it exists, find its value. **(5+5)**
6	Suppose $\lim_{n\to\infty} f_n(x) = f(x)$ pointwise,$x \in E$ and $M_n = sup_{x\in E}\lvert f_n(x) - f(x)\rvert$ then explain that $f_n(x) \to f_n$ uniformly iff $M_n \to 0$ as $n \to \infty$.
7	Show that a sequence of continuous function defined on an interval [a, b], if $f_n \to f$ uniformly on $[a,b]$, then f is continuous on $[a,b]$.
	SECTION C – K4 (CO3)
	Answer any TWO of the following **(2 x 12.5 = 25)**
8	State and prove the generalized mean value theorem.
9	If $f_1, f_2 \in \mathcal{R}(\alpha)$ on $[a,b]$. Determine that $\int_a^b (f_1 + f_2)d\alpha = \int_a^b f_1 d\alpha + \int_a^b f_2 d\alpha$.
10	Criticize that there exists a real continuous function on the real line which is nowhere differentiable.
11	Let $\{f_n\}$ be a uniformly convergent sequence with uniform limit f on $[a,b]$ and let f_n be integrable on $[a,b]$ $\forall\, n \in N$. Then determine that f is itself integrable and $\int_a^b f(x)dx = \lim_{n\to\infty} \int_a^b f_n(x)dx$
	SECTION D – K5 (CO4)
	Answer any ONE of the following **(1 x 15 = 15)**
12	Suppose f is continuous on $[a,b]$. $f'(x)$ exists at some point $x \in [a,b]$, g is defined on an interval I which contains the range of f and g is differentiable at $f(x)$. Determine that if $h(t) = g(f(t))$,$a \le t \le b$ then h is differentiable at x and $h'(x) = g'\big(f(x)\big)f'(x)$. Also Interpret the statement with $h(x) = sin\frac{1}{x}$, for every $x \neq 0$ in R.
13	a) Defend that every k-cell is compact. b) Let $f(x) = \begin{cases} x^2, & x \neq 1 \\ 0, & x = 1 \end{cases}$ $\lim_{x\to 1} x^2$ determine if limit exists.
	SECTION E – K6 (CO5)
	Answer any ONE of the following **(1 x 20 = 20)**
14	a) State and Demonstrate the necessary and sufficient condition for $f \in \mathcal{R}(\alpha)$. b) Derive the necessary and sufficient condition for uniform convergence.
15	Discuss and justify whether a uniformly continuous polynomial P_n is real for a continuous complex function f in $[a,b]$.

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