# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**M.C.A.** DEGREE EXAMINATION – **COMPUTER APPLICATIONS**

**FIRST** SEMESTER – **APRIL 2016**

**CA 1804 - DISCRETE STRUCTURES**

Date: 28-04-2016 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

## PART A

**Answer ALL Questions** **(10 X 2 = 20 Marks)**

1. What is tautology?
2. Write the truth table of $p \rightarrow q$.
3. What is least upper bound?
4. What is equivalence relation?
5. Define permutations and combinations.
6. When a function is said to be onto?
7. A connected graph contains Eular path iff it has exactly___ vertices of __ degree.
8. Mention the properties of Hamiltonian graph.
9. Define semigroup.
10. Define cosets.

## PART B

**Answer ALL Questions** **(5 X 8 = 40 Marks)**

11a. Constructing truth table for the following compound proposition:
$(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$

**(or)**

b. Prove the following equivalences by proving the equivalence of the duals:
$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$

12a. i. Define one-to-one function .
ii. Determine whether the following functions are one-to-one, onto, or one-to-one onto
(a) f:Z—>Z defined by $f(x)=x^2+5x+6$ (b) f:Z —>Z defined by $f(x)=x-5$

**(or)**

b. i. Define equivalence relation.
ii Let A = {0, 1, 2, 3}. Examine the following relations are equivalence relation
R = { (0,0), (1,1), (1,2), (2,1), (2,2), (3,3) }
R = { (0,0), (0,2), (2,0), (2,2), (2,3), (3,2), (3,3) }

13a. A Computer Science professor has 7 different programming books on a shelf, 3 of them deal with C++ and the other 4 with Java. In how many ways can the professor arrange these books on the shelf.
(i) if there are no restriction? (ii) if the languages to alternate?
(iii) if all the C++ books together and java books together? (iv) if all the C++ books together?

**(or)**

b. Find the number of integers between I and 150 (both inclusive) that are not divisible by any of the integers 2,3,and 5.

14a. i. Define the following terms in graph:
(a) degree of vertex (b) complete graph (c) Euler graph. (d) connected graph.

**(or)**

b. i. When are two graphs said to be isomorphic.

ii. Verify two graphs represented by their adjacency matrix which is given below is isomorphic.

$$A_1 = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \qquad A_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

15a.i. Define group.

ii. Show that the group (**G,** $+_5$) is a cyclic group where G ={0,1, 2, 3, 4}. What are its generators?

**(or)**

b. If * is defined on R such that a*b = a + b – ab for a, b ϵ R, show that (R, * ) is an abelian group.

## PART C

**Answer any TWO Questions** **(2 x 20 = 40 Marks)**

16a. i. What is Principal Disjunctive Normal Form? **(4 marks)**

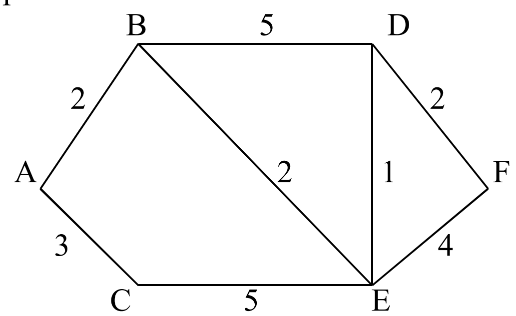
ii. Without constructing the truth table , find the principal disjunctive normal form of the following:

**(q v (p ^ r)) ^ ¬((p v r) ^ q)** **(6 marks)**

b. Let X = {1, 2, 3} , Y = {3, 5, 7, 9, 11}, Z = (4, 10, 16, 22, 28, 35 }
Let f: X → Y be defined by f(x) = 2x + 3 and g: Y → Z be defined by g(y) = 3y – 5. Find the composite functions i. **fog** ii**. gof** **(10 maks)**

17 a. State and prove Principle of Inclusion-Exclusion **(4 marks)**

b. Using Dijkstra algorithm, find the shortest path between vertex A and vertex F in the following graph. **(6 marks)**




18a. i. If S={1,2,3,6} and * is defined by a*b= lcm(a,b). Show that (S,*) is a monoid. What is identity element of S under *? **(4 marks)**

ii. If * is a binary operation on the set R of real numbers defined by a*b = a + b + 2ab then verify that (R, *) is a semigroup. Check whether it is commutative **(6 marks)**

b. Prove if f:G—>G' is a group homomorphism from (G, *) to (G', Δ) then

i. f(e) = e' where e and e' are identity elements of G and G'

ii. $f(a^{-1}) = (f(a))^{-1}$ for any a E G

iii. if H is a subgroup of G then f(H) = {f(h) / h E H} is a subgroup of G'. **(10 marks)**

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