LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **PHYSICS**

FIRST SEMESTER – **NOVEMBER 2019**

18/17/16PPH1MC04/PH 1820/PPH 1504 – MATHEMATICAL PHYSICS - I

Date: 07-11-2019   Dept. No.   Max. : 100 Marks
Time: 01:00-04:00

PART A

ANSWER **ALL** THE QUESTIONS   **10 x 2 = 20 Marks**

1. Write the algorithm of Runge-kutta method of solving 1st order differential equation $y' = y - x, \ y(0) = 1$.
2. Sketch the graph $y = \cos x$.
3. What are equipotential surfaces? Write its characteristic equation.
4. Show that $e^{i\alpha}$ is an operator.
5. Find the norm of $(1, 7, -2)$ in R^3 with standard inner product.
6. Define the terms positivity and point of symmetry of vector spaces.
7. Obtain an expression for $P_2(x)$ where 'P' stands for Legendre polynomials.
8. Write the orthogonality relation of Bessel's polynomials..
9. Prove that $\delta_j^i \delta_k^j = \delta_k^i$
10. State Hooke's law.

PART B

ANSWER ANY **FOUR** QUESTIONS   **4 x 7.5 – 30 Marks**

11. Compute the real root of $\sqrt[3]{18}$.
12. Derive Cauchy-Riemann conditions for a function to be analytic.
13. Show that the vectors u and v of a Euclidean space are orthogonal if and only if $\ u + v\|^2 = \|u\|^2 + \|v\|^2$.
14. i) Evaluate $\int_0^1 \frac{dx}{\sqrt{-\ln x}}$ using the knowledge of special functions.

    ii) Evaluate $\int_0^\infty x^3 e^{-x} \, dx$
15. i) Show that the sum of two tensors of the same order and type is again a tensor of the same order and type as the given tensor.

    ii) Show that a symmetric tensor of the second order has only $\frac{n(n+1)}{2}$ different components.
16. Using, Newton-Raphson method, evaluate $\overline{18}$.

PART C

ANSWER ANY **FOUR** QUESTIONS   **4 x 12.5 = 50 Marks**

17. Find the root of the equation $2x - log_{10}x = 7$, using Newton – Raphson method.
18. State and prove Cauchy's theorem.
19. Let v be the vector space of all polynomials in an determinate x, over the real field R of degree almost 2. In v, if we define inner product by $f(x), g(x) \in v \ as \ <f, g> = \int_{-1}^{+1} f(x)g(x)dx$ starting from $1, x, x^2$ of v obtain orthonormal basis.
20. Prove that $\int_{-1}^1 [P_n(x)^2]dx = \frac{2}{2n+1}$
21. Find the components of Euclidean metric tensor and obtain the expression for the line element in cylindrical coordinates.
22. Evaluate $\int_0^{2\pi} \frac{d\theta}{13 + 5\sin\theta}$ using contour integration.

@@@@@@

1