



**LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034**

**B.Sc. DEGREE EXAMINATION – ADV. ZOO., MATHS, PHYSICS, PLANT BIO., & BIO.**

**FOURTH SEMESTER – APRIL 2015**

**ST 4209 - MATHEMATICAL STATISTICS**

Date : 25/04/2015

Dept. No.

Max. : 100 Marks

Time : 09:00-12:00

**Section A**

**Answer all the questions**

**10 x 2 = 20 Marks**

1. Write the sample points when three fair coins are flipped simultaneously.
2. When three events are said to be mutually independent ?
3. If  $f(x) = e^{-x}$ ,  $x > 0$  find  $E(X)$ .
4. Show that standard normal distribution has mean 0 and variance 1.
5. If 10 unbiased coins are tossed simultaneously find the probability of getting at least 8 heads.
6. Write any four characteristics of normal distribution.
7. Define marginal and conditional distributions.
8. Define F distribution.
9. When an estimator is called unbiased ?
10. Define a simple and composite hypothesis.

**Section B**

**Answer any five questions**

**5 x 8 = 40 Marks**

11. State and prove Boole's inequality.
12. If  $P(A) = \frac{1}{2}$ ,  $P(B) = \frac{1}{4}$  and  $P(A \cap B) = \frac{1}{6}$  find (i)  $P(A | B)$  (ii)  $P(B | A)$   
(iii)  $P(A | B^c)$  (iv)  $P(A^c | B)$
13. Show that under certain conditions binomial tends to Poisson distribution.
14. If  $f(x) = 6x(1-x)$ ,  $0 < x < 1$ , zero elsewhere, compute  $P(\mu - 2\sigma < X < \mu + 2\sigma)$ .
15. Derive the moment generating function of binomial distribution and hence find mean and variance.
16. State and prove Chebyshev's inequality.
17. If  $X_1, X_2, \dots, X_n$  is a random sample from  $N(0, \theta)$ ,  $\theta > 0$  find the maximum likelihood estimator of  $\theta$ .
18. If  $f(x_1, x_2) = 12x_1x_2(1-x_2)$ ,  $0 < x_1 < 1$ ,  $0 < x_2 < 1$ , zero elsewhere find  
(i) the marginal distributions of  $X_1$  and  $X_2$ .  
(ii) the conditional distribution of  $X_1$  given  $X_2 = x_2$  and  $X_2$  given  $X_1 = x_1$ .

### Section C

Answer any two questions

2 x 20 = 40 Marks

19. (a) If two fair dice are thrown simultaneously and sum is noted, find the probability that the sum is (a) 5 (b) less than 7 (c) greater than 9 (d) between 6 and 10 inclusive.
- (b) Consider 3 urns. Urn 1 contains 5 red 6 blue and 7 black marbles. Urn 2 contains 6 red 8 blue and 4 black marbles. Urn 3 contains 7 red 5 blue and 6 black marbles. The probabilities of selecting the urns are respectively  $1/3$ ,  $1/2$  and  $1/6$  respectively. An urn is chosen at random and 3 marbles are chosen from it. They are found to be 1 blue and 2 black marbles. What is the probability that the chosen marbles have come from Urn 1, Urn 2 or Urn 3.

(8+12)

20. (a) Derive the moment generating function of normal distribution.

(b) If  $X$  is  $N(\mu, \sigma^2)$  so that  $P(X < 89) = 0.90$  and  $P(X < 94) = 0.95$  find  $\mu$  and  $\sigma^2$ . (10 +10)

21. If  $f(x, y) = x+y$   $0 < x < 1$ ,  $0 < y < 1$ , zero elsewhere find

(a) the conditional mean and variance of  $Y$  given  $X = x$ .

(b) the correlation coefficient of  $X$  and  $Y$ . (10+10)

- 22.(a) Derive the probability density function of Student's t distribution.

(b) Derive the mean and variance of F distribution. (10+10)

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