LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – **APRIL 2016**

ST 1503/ST 1501 – PROBABILITY AND RANDOM VARIABLES

Date: 03-05-2016 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

PART – A

Answer ALL questions: **(10 x 2 = 20)**

1. Define probability.
2. Define mutually exhaustive events with an example.
3. Write down the empirical definition of probability.
4. What is the probability of getting 9 cards of the same suit if 13 cards are drawn from a pack of cards?
5. Define independent events.
6. Define conditional probability.
7. State the multiplication law.
8. Prove that $P(\bar{A}) = 1 - P(A)$.
9. Define random variables.
10. Define continuous random variables.

PART – B

Answer any FIVE questions: **(5 x 8 = 40)**

11. State and prove addition theorem of probability for 2 events.
12. Four cards are drawn at random from a pack of 52 cards. Find the probability that
    i) They are a king, a queen, a jack and an ace.
    ii) Two are kings and two are queens.
    iii) Two are black and two are red.
    iv) There are two cards of hearts and two cards of diamonds.
13. From a city population, the probability of selecting i) a male or a smoker is 7/10, ii) a male smoker is 2/5, and iii) a male, given that a smoker is already selected is 2/3. Find the probability of selecting a) a non-smoker, b) a male and c) a smoker, given that a male is first selected.
14. Prove that for any three events A, B and C, P(AUB|C) = P(A|C) + P(B|C) – P(A∩ B|C).
15. A random variable X has the following probability function:

Values of X, x:	0	1	2	3	4	5	6	7
p(x):	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + 1$

    Find (i) k (ii) P(X < 6) and P(X ≥ 6) (iii) Find 'a' such that P(X≤ a) > ½ (iv) distribution function.
16. Let A and B be two events such that P(A) = $\frac{3}{4}$ and P(B) = $\frac{5}{8}$, show that a) P(AUB) ≥ $\frac{3}{4}$, and b) $\frac{3}{8} \le P(A \cap B) \le \frac{5}{8}$.
17. A letter of the English alphabet is chosen at random. Calculate the probability that the letter so chosen i) is a vowel, ii) precedes m and is a vowel, iii) follows m and is a vowel.
18. If X has the p.d.f f(x) = $\begin{cases} e^{-x}, & 0 < x < \infty \\ 0, & otherwise \end{cases}$, find E(X) and V(X) .

PART – C

Answer any TWO questions: **(2 x 20 = 40)**

19. a) Let X and Y be continuous random variables, Show that E(X+Y) = E(X) + E(Y). (10 Marks)

b) Two unbiased dice are thrown. Find the probability that: i) both the dice show the same number, ii) the first die shows 6, iii) the total of the numbers on the dice is 8, iv) the total of the numbers on the dice is greater than 8, v) the total of the numbers on the dice is 13, and vi) the total of the numbers on the dice is any number from 2 to 12, both inclusive. (10 Marks)

20. a) An urn contains 6 white, 4 red and 9 black balls. If 3 balls are drawn at random, find the probability that: i) two of the balls drawn are white, ii) one is of each colour, iii) none is red, iv) at least one is white. (10 Marks)

b) If the letters of the word 'REGULATIONS' be arranged at random,
i) What is the chance that there will be exactly 4 letters between R and E?
ii) What is the probability that four S's come consecutively in the word 'MISSISSIPPI'? (10 Marks)

21. a) State and prove Baye's theorem. (10 Marks)

b) The contents of urns I, II and III are follows:
1 white, 2 black and 3 red balls, 2 white, 1 black and 1 red balls, and 4 white, 5 black and 3 red balls. One urn is chosen at random and two balls drawn from it. They happen to be white and red. What is the probability that they come from urns I, II and III? (10 Marks)

22. a) Let X be a continuous random variable with p.d.f. given by:

$$f(x) = \begin{cases} kx, & 0 \le x < 1 \\ k, & 1 \le x < 2 \\ -kx + 3k, & 2 \le x < 3 \\ 0, & Otherwise \end{cases}$$

i) Determine the constant k, ii) Determine E(X) and
iii) What is the probability that X > 1.5? (10 Marks)

b) State and prove Chebyshev's inequality. (10 Marks)

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