LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – **APRIL 2016**

ST 1820 – ADVANCED DISTRIBUTION THEORY

Date: 28-04-2016 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

SECTION - A

Answer **ALL** questions. Each carries **TWO** marks. (10 x 2 = 20 marks)

1. Show that X is a random variable, when it denotes the number of heads obtained if a coin is tossed twice.
2. Write the pdf of truncated binomial, left truncated at '0' and derive its mgf.
3. Prove that the truncated Poisson distribution, truncated at zero, is a power series distribution.
4. State lack of memory property and show that Geometric distribution satisfies it.
5. Obtain the distribution of $\frac{1}{x}$, when X follows Lognormal.
6. Find the distribution of 2X, when X follows Inverse Gaussian.
7. Obtain the pgf and hence mgf of power-series distribution.
8. Let (X_1, X_2) follow BB (n, p_1, p_2, p_{12}). Write the marginal distributions of X_1 and X_2,
9. State and prove additive property of bivariate Poisson distribution.
10. Let X follow B(2, θ), θ = 0.1, 0.2, 0.3 and let θ follow discrete uniform. Derive the mean of the compound distribution.

SECTION - B

Answer any **FIVE** questions. Each carries **EIGHT** marks. (5 x 8 = 40 marks)

11. Obtain the decomposition of the distribution function F of a random variable X given by

$$F(x) = \begin{cases} 0, & x < 2 \\ \left(\frac{2}{3}\right)x - 1, & 2 \le x < 3 \\ 1, & 3 \le x < \infty \end{cases}$$

    Hence find the mgf of X.
12. State and prove a characterization of Poisson distribution through pdf.
13. Define inverse Gaussian distribution and find its mgf.
14. Show that $BVP(\lambda_1, \lambda_2, \lambda_{12})$ is the limiting distribution of $BB(n, p_1, p_2, p_{12})$.
15. Derive the regression equations associated with bivariate binomial distribution.
16. Let (X_1, X_2) follow BVN $(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. Obtain the conditional distribution of $X_2 \mid X_1 = x_1$.
17. Obtain a characterization of exponential distribution through failure rate function.
18. Define non-central't' distribution and find its pdf.

SECTION – C

Answer any TWO questions. Each carries TWENTY marks. (2 x 20 = 40 marks)

19. (a) Let $X_1, X_2, \ldots, X_n$ be iid non-negative integer valued random variables. Prove that X_1 is Geometric iff $\text{Min}\{X_1, X_2, \ldots, X_n\}$ is Geometric. (10)

    (b) Let $(X_1, X_2) \sim BB(n, p_1, p_2, p_{12})$. Show that $(X_1 \mid X_2 = x_2) \stackrel{d}{=} (U_1 + V_1)$, where $U_1 \sim B\left(n - x_2, \frac{p_{12}}{p_1 + p_{12}}\right)$, $V_1 \sim B\left(x_2, \frac{p_{12}}{p_2 + p_{12}}\right)$ and U_1 is independent of V_1. (10)

20(a) Find the cgf of power series distribution. Hence obtain the recurrence relation satisfied by the cumulants. (10)

(b) Let X_1 and X_2 be two independent Normal variables with the same variance. State and prove a necessary and sufficient condition for two linear combinations of X_1 and X_2 to be independent. (10)

21. (a) If X follows IG (μ, λ), then prove that $((X - \mu)^2) / (\mu^2 X)$ follows $\chi^2(1)$. (10)

(b) Show that mean > median > mode for a Log-Normal distribution. (10)

22. (a) Derive the mgf of (X_1, X_2) at (t_1, t_2), when $(X_1, X_2) \sim BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. (10)

(b) Let X_1, X_2, X_3 be independent normal variables such that $E(X_1) = 1$, $E(X_2) = 3$, $E(X_3) = 2$ and $V(X_1) = 2$, $V(X_2) = 2$, $V(X_3) = 3$. Verify the independence of the following pairs:

(i) $X_1 + X_2$ and $X_1 - X_2$

(ii) $X_1 + X_2 - 2X_3$ and $X_1 - X_2 + 2X_3$

(iii) $2X_1 + X_3$ and $X_2 - X_3$. (10)

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