LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – **APRIL 2016**

ST 2502/ST 2501/ST 2500 – STATISTICAL MATHEMATICS - I

Date: 23-04-2016 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART – A

Answer ALL questions: **(10 x 2 = 20 marks)**

1. Define function.
2. Define convergent.
3. Define absolute convergence.
4. Define pgf of random variable.
5. Define Rolle 's Theorem.
6. Define Taylor's expansion of a function about x = a.
7. Define orthogonal vectors.
8. What is meant by linear dependence.
9. Define orthogonal matrix.
10. Define positive definite matrix.

PART – B

Answer any FIVE questions: **(5 x 8 = 40 marks)**

11. Prove that the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$ is convergent.
12. State and prove the ratio test.
13. State and prove Rolle 's Theorem.
14. A continuous random variable has the pdf given by f(x) = $\begin{cases} kx(1-x); 0 < x < 1 \\ 0 \quad\quad otherwise \end{cases}$. Determine the constant k and also obtain mean and variance.
15. If A = $\frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$ then show that A is an orthogonal matrix.
16. Find the MGF of the random variable whose moments are $\mu_r' = (r+1)!\,2^r$.
17. Find the rank of the matrix A = $\begin{bmatrix} 1 & 5 & 6 \\ 0 & 0 & 0 \\ 2 & 4 & 5 \end{bmatrix}$.
18. Prove that $E[E(X/Y)]$ = E(X).

PART – C

Answer any TWO questions: **(2 x 20 = 40 marks)**

19. a) Show that every bounded sequence is a convergent. (8 Marks)

b) For the joint distribution $f(x, y) = \begin{cases} \frac{9}{4} - x - y; & 0 \le x \le 2; 0 \le y \le 2 \\ 0; & elsewhere \end{cases}$. Obtain the marginal and conditional distribution. (12 Marks)

20. a) State and prove Mean value Theorem. (10 Marks)

b) If $y = \frac{x}{\sqrt{1+x^2}}$, prove that $x^3 \frac{dy}{dx} = y^3$. (10 Marks)

21. A random variable X has the following probability function.

X	0	1	2	3	4	5	6	7
P(x)	0	K	2k	2k	3k	k^2	$2 k^2$	$7k^2 + k$

i) Find k
ii) Evaluate a) P(X<6) b) P(0<X<5)
iii) If $P(X \le k) > \frac{1}{2}$, find the minimum value of k
iv) Determine the distribution function of X.

22. a) Given $f(x, y) = \begin{cases} xe^{-x(1+y)}, & x \ge 0, y \ge 0 \\ 0; & Otherwise \end{cases}$. Find E(XY) and E($y$ / x). Show that E(Y) does not exist. (10 Marks)

b) Find the inverse of the matrix $A = \begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix}$. (10 Marks)

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