LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – **APRIL 2016**

ST 2814 - ESTIMATION THEORY

Date: 16-04-2016 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

SECTION – A

Answer ALL the questions **(10 x 2 = 20)**

1. Give an example to prove that an unbiased estimator need not be unique.
2. State the different approaches to identify UMVUE
3. Define Sufficient Statistic.
4. Let X be random variable with pmf: $P(X=1)=\frac{\theta}{2}$, $P(X=2)=1-\frac{\theta}{2}$. Find the Fishers Information contained in X?
5. Find which one of the following is ancillary when a random sample X_1, X_2 is drawn from N(μ,1).
   (a) X_1/X_2 (b) X_1+X_2 (c) X_1 - X2 (d) $2X_1-X_2$
6. Let $X_i \sim N(\mu,\sigma^2),\ i=1,2,...,n,\quad \mu\in R,\ \ \sigma>0,$. Show that CRLB for estimating μ^2 is $\frac{4\mu^2}{n}$.
7. Explain the concept of likelihood function.
8. What is exponential class of family?
9. Suggest an MLE for P[X=0] in the case of P(θ), θ>0.
10. Define CAN estimator.

SECTION – B

Answer any FIVE questions **(5 x 8 = 40)**

11. State and Prove a necessary and sufficient condition for an estimator to be UMVUE using uncorrelatedness approach.
12. If δ_0 be a fixed member of U_g, then prove that $U_g=\{\delta_0+u \mid u\in U_0\}$.
13. Let $X_1,X_2,\ldots,X_n$ be a random sample of size n from $U[0,\theta],\ \theta>0$. Find the Sufficient Statistic for θ.
14. Let $X_1,X_2,\ldots,X_n$ be a random sample from $N(0,\theta^2)$. Obtain the Cramer – Rao lower bound for estimating θ^2.
15. Show that the family of $B(n,\theta)$, $0<\theta<1$ is Complete.
16. Let $X_1,X_2,\ldots,X_n$ be a random sample of size n from $P(\theta),\ \theta>0$. Obtain MVBE of θ and suggest MVBE of $a\theta+b$, where a and b are constants such that $a\neq 0$.
17. State and Establish Basu's theorem
18. Let $X_1,X_2,\ldots,X_n$ be a random sample from $N(\mu,1),\ \mu\in R$. Let μ have the prior distribution $N(0,1)$. Find the Bayes estimator of μ.

SECTION – C

Answer any TWO questions **(2x 20 = 40)**

19. (a) Let X be a discrete r.v. with $P(x;\theta) = \begin{cases} \theta & , x=-1 \\ (1-\theta)^2\,\theta^x & , x = 0,1,2,\ldots \end{cases}$

Find all the unbiased estimators of 0.

(b) Let $X_i \sim U(0,\theta),\ \theta > 0,\ i = 1,2,\ldots,n$. Find UMVUE of $\dfrac{n}{n+1}\theta$ **(10+10)**

20. (a) Explain completeness and boundedly completeness with an illustration.

(b) State and establish Lehmann-Scheffe theorem. **(10+10)**

21. (a) Let (X_i,Y_i) , i=1,2,…,n be a random sample from ACBVE distribution with pdf

$$f(x,y) = \{(2\alpha+\beta)(\alpha+\beta)/2\}\exp\{-\alpha(x+y)-\beta\max(x,y)\},\ x,y>0\ .$$

Find MLE of α and β.

(b) MLE is not consistent – Support the statement with an example. **(10+10)**

22. (a) “Blind use of Jackknife method” – Illustrate with an example.

(b) Explain Baye’s estimation with an example. **(10+10)**

****