LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – **APRIL 2016**

ST 2962 – MODERN PROBABILITY THEORY

Date: 27-04-2016 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

SECTION A

Answer ALL of the following. **(10x2=20 marks)**

1. Can the union of two fields be a field? Explain.
2. Define: Monotone Field.
3. Define: Discrete Probability Space.
4. Give an example for the minimal field containing the class itself.
5. Define Mixture of Distributions.
6. Let X be a continuous Gamma Variate, derive, $M_x(\theta)$.
7. Derive the Harmonic Mean of Beta Distribution of first kind.
8. Explain almost sure convergence.
9. When will you say, a random variable is centered at some constant c and its expectation?
10. Explain law of large numbers?

SECTION B

Answer any FIVE from the following **(5x8=40 marks)**

11. Let ξ_i be the class of all intervals of the form (a, b), $(a<b)$ a,b $\in$ R, but arbitrary. Then P.T. $\sigma(\xi_i) = \mathcal{B}$.
12. Explain: Induced Probability Space with an example.
13. Show that Standard Gamma variate converges in distribution to Standard Normal variate.
14. If $\sum \sigma^2_n < \infty$ then, prove that, $\sum(X_k - E(X_k))$ converges in probability.
15. State and prove the properties of Expectation of Non negative random variables.
16. If $X_n \xrightarrow{p} X$ then, show that there exists a subsequence $\{X_{nk}\}$ of $\{X_n\}$ which converges a.s. to X.
17. State and prove the properties of characteristic function.
18. Explain in detail the applications of Central limit theorem.

SECTION C

Answer any TWO from the following **(2x20=40 marks)**

19. i) Prove that The intersection of arbitrary number of fields is a field. (10)

ii) Prove that $X_n \xrightarrow{p} c$ implies that $F(X_n) \longrightarrow 0$ for $x<c$, $F(X_n) \longrightarrow 1$ for $x \geq c$ and conversely. (10)

20. i) Let $X_n \xrightarrow{L} X, Y_n \xrightarrow{L} c$, then P.T.

a. $X_n + Y_n \xrightarrow{L} X + c$

b. $X_n Y_n \xrightarrow{L} cX$

c. $X_n / Y_n \xrightarrow{L} X / c$. (10)

ii) Let $\{x_n\}$ be a sequence of i.i.d. r.v.'s with characteristic function $\varphi(u)$. Then $S_n/n \xrightarrow{p} E(X)$. (10)

21. i) Derive the Kolmogorov Inequalities. (10)

ii) State and prove the necessary and sufficient condition for a series of random variables to converge a.s. (10)

22. i) State and prove Liapounov's theorem. (12)

ii) Explain Central limit theorem as a generalisation of law of large numbers. (8)

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