LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – **APRIL 2016**

ST 3506 – MATRIX AND LINEAR ALGEBRA

Date: 02-05-2016 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART A

Answer ALL the questions. **(10 x 2 =20 Marks)**

1. Define equality of matrices.
2. Define a symmetric matrix.
3. Define singular matrix with an example.
4. Find rank of A, where A is given by

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 2 & 2 \\ 1 & 2 & 2 \end{bmatrix}.$$

5. How do you define vector space?
6. Define basis and dimension in a vector space.
7. When a transformation is said to be one to one and onto?
8. Define a unitary matrix.
9. Find the characteristic root of the matrix.

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}.$$

10. Show that if λ is a characteristic root of a matrix A, the λ^k is the characteristic root of A^k .

PART B

Answer any FIVE questions. **(5 x 8 = 40marks)**

11. If A is any nxm matrix such that AB and BA are both defined. Show that B is an mxn matrix.
12. If A is any square matrix, then show that $A + A'$ is symmetric and $A - A'$ is Skew - symmetric.
13. Prove that every invertible matrix possesses a unique inverse.
14. Show that the matrix $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix}$ equivalent to I_3 by finding the rank.
15. Prove that a basis of a subspace S, can always be selected out of a set of vectors which span S.
16. Show that the set of 3 vectors $X_1 = (1\ 0\ 0)$, $X_2 = (0\ 1\ 0)$ and $X_3 = (0\ 0\ 1)$ is linearly independent.
17. (a) State the properties of linear transformation.
    (b) Prove that a linear transformation $L : V_n \rightarrow W_n$ is completely determined if it is specified at the basis element V_n.
18. Determine eigen values and eigen vectors of the matrix.

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}.$$

PART C

Answer any TWO questions. **(2 x 20 =40 Marks)**

19.(a) If A and B are two matrices such that AB = BA then show that for every positive integer n, by induction

(i). $AB^n = B^nA$ (ii) $(AB)^n = A^n B^n$.

(b) Let A and B be two square matrices of order n and λ be a scalar. Then prove that

(i) $Tr(\lambda A) = \lambda Tr(A)$

(ii) $Tr(A + B) = Tr(A) + Tr(B)$.

20. (a) Find the rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}.$$

(b) Show that

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a).$$

21. (a) Let $X_1 = (1, 1, -1)$, $X_2 = (4,1,1)$ and $X_3 = (1,-1,2)$ be a basis of R_3 and let $L: R_3 \to R_2$ be the linear transformation such that $LX_1 = (1, 0)$, $LX_2 = (0,1)$, $LX_3 = (1,1)$. Find L.

(b) Solve the following equations by cramer's rule.

$$2x - y + 3z = 9$$
$$x + y + z = 6$$
$$x - y + z = 2.$$

22. (a) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}.$$

(b). State and prove Cayley- Hamilton theorem.

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