LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – **APRIL 2016**

ST 4201 - MATHEMATICAL STATISTICS

Date: 28-04-2016 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

Section -A

Answer all the questions : **10 X 2 = 20 marks**

1. Write the sample space for tossing four fair coins.
2. If three fair dice are flipped, find the probability of the sum to be either 17 or 18.
3. State two properties of distribution function.
4. Define moment generating function.
5. If ten unbiased coins are tossed , find the probability of getting at least two heads.
6. Define chi-square distribution with n degrees of freedom.
7. Let X have the probability mass function $p(x) = (1/2)^x$, x=1,2,3,…, zero elsewhere.
   Find the probability mass function of $Y = X^3$.
8. Define marginal and conditional distributions.
9. Provide the sufficient conditions for consistency of an estimator.
10. Define simple and composite hypothesis.

Section –B

Answer any five questions: **5 X 8 = 40 marks**

11. (a) State addition theorem on probability for n events. (2 marks)
    (b) State and prove Bayes' theorem.(6 marks)
12. If P(A) = 1/3 , P(B) = 1/5 and $P(A \cap B) = 1/9$, find (i) $P(A \mid B^C)$ (ii) $P(A^C \mid B)$
    (iii) $P(A^C$ □ $B^C)$ and (iv) $P(A^C \mid B^C)$. (4 X 2 =8 marks)
13. Show that under certain conditions binomial distribution tends to Poisson.
14. Derive mean and variance of rectangular distribution over [a,b].
15. State and prove Boole's inequality.
16. Find mean deviation from mean for normal distribution.
17. Show that the random variables X_1 and X_2 with joint p.d.f.
    $f(x_1,x_2) = 12\, x_1x_2(1-x_2)$, $0 < x_1 < 1$, $0 < x_2 < 1$, zero elsewhere, are stochastically independent.
18. If $X_1, X_2, \ldots, X_n$ is a random sample from normal distribution with mean θ_1 and variance θ_2,
    find the maximum likelihood estimators of θ_1 and θ_2.

Section-C

Answer any two questions: **2 X 20 =40 marks**

19. (a) State and prove Chebyshev'sinequality.(8 marks)

(b) Derive mean and variance of beta distribution of first kind.(12 marks)

20. (a) Derive the moment generating function of normal distribution.(8 marks)

(b) Let the marks obtained in a certain examination follow the normal distribution with mean 45 and standard deviation 10. If 1,000 students appeared at the examination , calculate the number of students scoring:(i) less than 40 marks(ii) more than 60marks (iii) between 40 and 50 marks. (12 marks)

21. Let X_1 and X_2 have the joint p.d.f. $f(x_1,x_2) = x_1 + x_2$, $0<x_1<1$,$0<x_2<1$, zero elsewhere.

Find the conditional mean and variance of X_1 given $X_2 = x_2$, $0<x_2<1$ and X_2 given $X_1=x_1$, $0<x_1<1$.

22. Derive the probability density function of F distribution. Also find mean and variance.

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