LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

FOURTH& FIFTH SEMESTER – APRIL 2017

ST 4503 / ST 5504 - ESTIMATION THEORY

Date: 21/04/2017 Dept. No. Max. : 100 Marks
09:00-12:00

SECTION – A

Answer **ALL** questions. **10 X 2 = 20**

1. Define Consistent estimator. Give an example.
2. State any two regularity conditions.
3. Define UMVUE.
4. When $X_1, X_2, X_3, \ldots\ldots\ldots, X_n$ are random samples from Binomial $(10,\theta)$, suggest a sufficient statistic.
5. Describe the Method of Maximum Likelihood estimation.
6. State the Least Square estimator of β_1, in the model $Y = \beta_0 + \beta_1 X + \varepsilon$
7. Define risk function.
8. State a possible Prior distribution for the parameter λ in case of Poisson distribution with mean λ.
9. State the 95% confidence interval for μ, when a random sample of size 'n' is drawn from N(μ , 1).
10. State Rao-Blackwell theorem.

SECTION – B

Answer Any **FIVE** questions. **5 X 8 = 40**

11. Derive an Unbiased estimator of λ in a Poisson distribution, based on a random sample of size 'n'.
12. If $X_1, X_2, X_3, \ldots\ldots\ldots, X_n$ is random sample of size 'n' fromBinomial(1, θ) , $\theta \in (0,1)$, then show that $\sum_{i=1}^{n} X_i$ is a sufficient statistic for θ, using Neyman Factorization theorem.
13. Define completeness of an estimator and verify whether $\bar{x}$ is a complete estimator in case of a random sample of size 'n' fromN(θ ,1) , $\theta \in R$.
14. Explain the Method of Moment estimation.
15. State any four properties of MLE.
16. Describe Bayes estimation procedure.
17. What is a conjugate prior? Give an example.
18. State and prove Neyman –Fisher factorization theorem.

SECTION – C

Answer any **TWO** questions. **2 X 20 = 40**

19. a. State and prove Cramer-Rao inequality. **[12]**

b. Show that the family of the Binomial distributions { B(n, θ) , $\theta \in (0,1)$, n- known } is complete. **[8]**

20. a. State and prove Lehmann- Scheffe theorem. **[10]**

b. Show that UMVUE is unique, when it exists. **[10]**

21. a. Explain the concept of estimation by the method of minimum chi-square. **[8]**

b. For a random sample $X_1, X_2, X_3 .... X_n$ from U(α, β), $\alpha < \beta$, $\alpha, \beta \in \mathcal{R}$, obtain method of moment estimators of α and β. **[12]**

22. a. Let $X_1, X_2, X_3, ..., X_n$ be a random sample from Binomial(m, θ) , $\theta \in (0,1)$, m-known and let Beta(α, β) be the prior distribution for θ. Find the Bayesian estimator for θ. **[12]**

b. If $X_1, X_2, X_3, ..., X_n$ and $Y_1, Y_2, Y_3, ..., Y_m$ are random samples of size 'n' and 'm' respectively from N(μ_1, σ^2) and N(μ_2, σ^2), derive the (1-α)100% confidence interval for their difference in means. **[8]**

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